Published by The College of Science & Information Technology (COSIT), TASUED, Vol. 19, No. 1, pp. 331-338.

COSIT, TASUED
Journal of Science and Information Technology (JOSIT)

Transform Methods for N^{th} Order Linear ODEs: A Comparative Evaluation of Elzaki and Laplace Approaches

Amurawaye F. F.

Department of Mathematical Sciences, Tai Solarin University of Education, Ogun State, Nigeria.

Corresponding Author: amurawayeff@tasued.edu.ng

Abstract

In this paper, we present a comparative study of the application of a relatively new integral transform similar in spirit with Laplace transform, the Elzaki transform in solving linear ODE with constant coefficients. Different problems were considered using both techniques by illustrating their respective solution method. It was revealed that the Elzaki transform is powerful and efficient in solving nth order ODE with easier simple procedures.

Keywords: Elzaki transform, Laplace transform, Differential equation, N^{th}-Order

INTRODUCTION

Modelling of numerous physical issues in the real world leads to the emergence of rigid systems of ordinary differential equations (ODEs). The scientific community is very interested in the solutions to such systems. For the solution of such systems, a number of numerical, analytical, and semi-numerical strategies have been put forth. Linear ODE are fundamental tools in modelling analysis across disciplines such as physics, engineering and applied Mathematics. One of the classical methods for solving such equations is the Laplace transform.

Integral transforms are important mathematical tools that help simplify the process of solving differential equations. One of the most widely used is the Laplace Transform, introduced by Pierre-Simon Laplace in 18th century. Over time, it has become central to applications in engineering, physics, and control theory (Doetsch, 1974; Debnath & Bhatta, 2007). Its strength lies in reducing complex initial value problems into forms that are much more manageable. An alternative integral transform method is the Elzaki Transform which is relatively new method that addresses lengthy algebraic manipulations that might sometimes arises in the use of the Laplace transform. This relatively new method has shown promise in simplifying the solution process for both ordinary and fractional differential equations, providing results with reduced computational effort (Elzaki, 2011). In some cases, it offers advantages over both the Laplace and Sumudu transforms (Belgacem & Karaballi, 2006).

The comparative study of the third order convergence Numerical method (FS), Adomain Decomposition Method (ADM), and successive approximation method (SAM) in the context of exact solution of ODE were presented by Fadugba et.al (2020). Assabaai & Ahmed (2023) numerically solved singular linear ordinary differential equations (SLODEs) of high orders using the collocation method on the NBI polynomials. William (2021) presented the Laplace transform as an alternative general method for solving linear ordinary differential equations. Areo et al. (2020) proposed the derivation of a class of hybrid methods for solution of second order initial value problems

Cite as:

Amurawaye F. F. (2025). Transform Methods for N^{th} Order Linear ODEs: A Comparative Evaluation of Elzaki and Laplace Approaches. *Journal of Science and Information Technology (JOSIT),* Vol. 19, No. 1, pp. 331-338.

(IVP) in blockmode using it to investigate order, error constant, zero stability, consistency and convergence. Triag & Salih (2011) presented the connection between Elzaki transform and Laplace Transform. A family of matrix coefficient formulas for solving ordinary differential equations was studied by Shuenn-Yih (2022). Amurawaye et al. (2023) studied the application of Aboodh transform to solution of nth order differential equations and compared results with Laplace Transform. Triag & Salih (2011) worked on the Elzaki transform and ordinary differential equations. This article contribution show how both classical and modern approaches can complement each other in problem – solving.

MATERIALS AND METHODS

The Elzaki transform is a relatively recent integral transform introduced by Elzaki (2011). This transform has gained attention due to its effectiveness in solving linear and nonlinear differential equations particularly where traditional method maybe cumbersome.

Like the Laplace transform, the Elzaki transform is an integral transform defined for a function $F(t), t \geq 0$, as :

$$E[F(t)] = v^2 \int_0^\infty F(vt) e^{-t} = E(y), \quad v \in k_1, k_2$$

where $v \in k_1, k_2$

Elzaki transform of 1st Order ODE

Given $ay' + by = c, \quad y(0) = k$

By taking Elzaki transform of this equation,We have

$$a\left[\frac{1}{v}E(y) - vy(0)\right] + bE(y) = E(c)$$

$$\frac{a}{v}E(y) - avy(0) + bE(y) = E(c)$$

$$\frac{a}{v}E(y) - avk + bE(y) = E(c)$$

$$\frac{a}{v}E(y) + bE(y) = avk + E(c)$$

$$E(y)\left[\frac{a}{v} + b\right] = avk + E(c)$$

$$E(y)\left[\frac{a+bv}{v}\right] = avk + E(c)$$

$$E(y)\left[\frac{a+bv}{v}\right] = [avk + E(c)] \times \frac{v}{a+bv}$$

$$E(y) = \frac{av^2k + vE(c)}{a+bv} \qquad (1)$$

Taking the inverse Elzaki transform of equation (1) we get desired result.

Elzaki transform of 2nd Order ODE

$ay'' + by' + cy = d, \quad y(0) = k,$ and $y'(0) = P$

Taking Elzaki transform of this equation result into

$$a\left[\frac{1}{v^2}E(y) - y(0) - vy'(0)\right] + b\left[\frac{1}{v}E(y) - y(0)\right] + cE(y) = E(d)$$

$$\frac{a}{v^2}E(y) - avk - avP + \frac{b}{v}E(y) - bk + cE(y) = E(d)$$

$$E(y)\left[\frac{a}{v^2} + \frac{b}{v} + c\right] = ak + avP + bk + E(d)$$

$$E(y)\left[\frac{a+bv+cv^2}{v^2}\right] = ak + avP + bk + E(d)$$

$$E(y) = \frac{v^2[ak + avP + bk + E(d)]}{a+bv+cv^2} \qquad (2)$$

After necessary mathematical simplification and taking the inverse Elzaki transform (2), we arrived at exact solution.

Elzaki transform of 3rd Order Ordinary Differential Equation

$ay''' + by'' + cy' + dy = e, \quad y(0) = k, \quad y'(0) = P$ and $y''(0) = m$

$$a\left[\frac{1}{v^3}E(y) - \frac{y(0)}{v} - y'(0) - vy''(0)\right] + b\left[\frac{1}{v^2}E(y) - y(0) - vy'(0)\right]$$

$$+ c\left[\frac{1}{v}E(y) - y(0)\right]E(y) + dE(y) = E(e)$$

$$\frac{a}{v^3}E(y)-\frac{ak}{v}-aP-avm+\frac{b}{v^2}E(y)-bk-bvP+\frac{c}{v}E(y)-ck+dE(y)=E(e)$$

$$E(y)\left[\frac{a}{v^3}+\frac{b}{v^2}+\frac{c}{v}+d\right]=\frac{ak}{v}+aP+avm+bk+bvP+ck+E(e)$$

$$E(y)\left[\frac{a+bv+cv^2+dv^3}{v^3}\right]=\frac{ak+aPv+amv^2+bkv+bPv^2+ckv+vE(e)}{v}$$

$$E(y)=\frac{v^2\left[ak+aPv+amv^2+bkv+bPv^2+ckv+vE(e)\right]}{a+bv+cv^2+dv^3} \qquad (3)$$

Simplifying (3) and taking the inverse Elzaki transform, gives desired result.

N^{th} order ODE by Elzaki transform

Consider the n^{th} order ODE

$a_n\frac{d^ny}{dx^n}+a_{n-1}\frac{d^{n-1}y}{dx^{n-1}}+a_{n-2}\frac{d^{n-2}y}{dx^{n-2}}+\cdots+a\frac{dy}{dx}+a_0y=b(x)$ with the initial conditions $y(0)=k_0, y'(0)=k_1, y''(0)=k_2, y'''(0)=k_3, \ldots, y^{n-2}(0)=k_{n-2}$ and $y^{n-1}(0)=k_{n-1}$ (4)

Taking Elzaki transform of (4)

$$E\left(a_n\frac{d^ny}{dx^n}\right)=a_n\left(\frac{E(y)}{v^n}-\frac{y(0)}{v^{n-2}}-\frac{y'(0)}{v^{n-3}}-\frac{y''(0)}{v^{n-4}}-\frac{y'''(0)}{v^{n-5}}-\frac{y'^{v}(0)}{v^{n-6}}-\cdots-y^{n-1}(0)\right) \qquad (5)$$

$$E\left(a_{n-1}\frac{d^{n-1}y}{dx^{n-1}}\right)=a_{n-1}\left(\frac{E(y)}{v^{n-1}}-\frac{y(0)}{v^{n-3}}-\frac{y'(0)}{v^{n-4}}-\frac{y''(0)}{v^{n-5}}-\frac{y'''(0)}{v^{n-6}}-\frac{y'^{v}(0)}{v^{n-7}}-\cdots-y^{n-2}(0)\right) \qquad (6)$$

$$E\left(a_{n-2}\frac{d^{n-2}y}{dx^{n-2}}\right)==a_{n-2}\left(\frac{E(y)}{v^{n-2}}-\frac{y(0)}{v^{n-4}}-\frac{y'(0)}{v^{n-5}}-\frac{y''(0)}{v^{n-6}}-\frac{y'''(0)}{v^{n-7}}-\frac{y'^{v}(0)}{v^{n-8}}-\cdots-y^{n-3}(0)\right) \qquad (7)$$

…….

……..

…….

$$E\left(a\frac{dy}{dx}\right)=a\,E\left(\frac{dy}{dx}\right)=a\left(\frac{E(y)}{v}-vy(0)\right) \qquad (8)$$

$$E(a_0y)=a_0\,E(y) \qquad (9)$$

$$E(b(x))=E(b(x)) \qquad (10)$$

Simplifying and introducing the initial conditions, give result as;

$$\left[\frac{a_n}{v^n}+\frac{a_{n-1}}{v^{n-1}}+\frac{a_{n-2}}{v^{n-2}}+\cdots+\frac{a_1}{v^1}+\frac{a_0}{v^0}\right]E(y)-\left[\frac{a_n}{v^{n-2}}+\frac{a_{n-1}}{v^{n-3}}+\frac{a_{n-2}}{v^{n-4}}+\cdots+a_1v\right]k_0-\left[\frac{a_n}{v^{n-3}}+\frac{a_{n-1}}{v^{n-4}}+\frac{a_{n-2}}{v^{n-5}}+\cdots\right]k_1-\left[\frac{a_n}{v^{n-4}}+\frac{a_{n-1}}{v^{n-5}}+\frac{a_{n-2}}{v^{n-6}}+\cdots\right]k_2-\left[\frac{a_n}{v^{n-5}}+\frac{a_{n-1}}{v^{n-6}}+\frac{a_{n-2}}{v^{n-7}}+\cdots\right]k_3-\left[\frac{a_n}{v^{n-6}}+\frac{a_{n-1}}{v^{n-7}}+\frac{a_{n-2}}{v^{n-8}}+\cdots\right]k_4-[a_nk_{n-1}+a_{n-1}k_{n-2}+a_{n-2}k_{n-3}+\cdots]=E[b(x)] \qquad (11)$$

Equation (11) gives the solution of the n^{th} order ODE for Elzaki transform.

Laplace Transform of 1st order ODE.

$ay'+by=c, \quad y(0)=k$

Taking Laplace transform of the equation,

$$L[ay']+L[by]=L(c),$$
$$a[SL(y)-y(0)]+bL(y)=L(c)$$
$$aSL(y)-ak+bL(y)=L(c)$$
$$aSL(y)+bL(y)=L(c)+ak$$
$$L(y)[aS+b]=L(c)+ak$$
$$L(y)=\frac{L(c)+ak}{aS+b} \qquad (12)$$

Taking inverse Laplace transform, gives exact solution.

Laplace Transform of 2nd order ODE

$ay''+by'+cy=d, \quad y(0)=k,$ and $y'(0)=P$

Taking Laplace transform of this equation,

$$a[S^2L(y)-Sy(0)-y'(0)]+b[SL(y)-y(0)]+cL(y)=L(d)$$

$$aS^2L(y)-aSk-aP+bSL(y)-bk+cL(y)=L(d)$$

$$L(y)\left[aS^2+bS+c\right]=aSk+aP+bk+L(d)$$

$$L(y)=\frac{aSk+aP+bk+L(d)}{aS^2+bS+c} \qquad (13)$$

By Mathematical simplification of (13) and taking the inverse Laplace transform, gives result.

Laplace Transform of 3rd order ordinary differential equation

$ay'''+by''+cy'+dy=e,\quad y(0)=k,\quad y'(0)=P$ and $y''(0)=m$

Taking Laplace transform of this equation

$$L[ay''']+L[by'']+L[cy']+L[dy]=L(e)$$

$$a\left[S^3L(y)-S^2y(0)-Sy'(0)-y''(0)\right]+b\left[S^2L(y)-Sy(0)-y'(0)\right]+c\left[SL(y)-y(0)\right]+dL(y)=L(e)$$

$$aS^3L(y)-aS^2k-aPS-am+bS^2L(y)-bkS-bP+cSL(y)-ck+dL(y)=L(e)$$

$$L(y)\left[aS^3+bS^2+cS+d\right]=aS^2k+aPS+am+bkS+bP+ck+L(e)$$

$$L(y)=\frac{aS^2k+aPS+am+bkS+bP+ck+L(e)}{aS^3+bS^2+cS+d} \qquad (14)$$

By taking the inverse Laplace transform, we arrived at our desired result.

N^{th} order ODE by Laplace transform

Consider the nth order ODE

$a_n\frac{d^ny}{dx^n}+a_{n-1}\frac{d^{n-1}y}{dx^{n-1}}+a_{n-2}\frac{d^{n-2}y}{dx^{n-2}}+\cdots+a\frac{dy}{dx}+a_0y=b(x)$ with the initial conditions $y(0)=k_0,\ y'(0)=k_1,\ y''(0)=k_2,\ y'''(0)=k_3,\ \ldots,\ y^{n-2}(0)=k_{n-2}$ and $y^{n-1}(0)=k_{n-1}$ (15)

Taking Laplace of (15), we have

$$L\left[a_n\frac{d^ny}{dx^n}\right]=a_n\,L\left[\frac{d^ny}{dx^n}\right]=a_n[S^ny(s)-S^{n-1}y(0)-S^{n-2}y'(0)-\cdots-y^{n-1}(0)] \qquad (16)$$

$$L\left[a_{n-1}\frac{d^{n-1}y}{dx^{n-1}}\right]=a_{n-1}\,L\left[\frac{d^{n-1}y}{dx^{n-1}}\right]=a_{n-1}[S^{n-1}y(s)-S^{n-2}y(0)-S^{n-3}y'(0)-\cdots-sy^{n-3}(0)-y^{n-1}(0)] \qquad (17)$$

……

……

$$L\left[a\frac{dy}{dx}\right]=aL\left[\frac{dy}{dx}\right]=a\,[Sy(s)-y(0)] \qquad (18)$$

$$L[a_0y]=a_0L[y]=a_0y(s) \qquad (19)$$

If $b(x)$ is a constant, combining equations (16) to (19), simplifying and using initial conditions gives:

$$[a_nS^n+a_{n-1}S^{n-1}+a_{n-2}S^{n-2}+\cdots a_{n-n}S^{n-n}]Y(s)-[a_nS^{n-1}+a_{n-1}S^{n-2}+a_{n-2}S^{n-3}+\cdots a_{n-(n-1)}S^{n-n}]k_0-[a_nS^{n-2}+a_{n-1}S^{n-3}+a_{n-2}S^{n-4}+\cdots a_{n-(n-2)}S^{n-n}]k_1-\cdots-[a_nk_{n-2}+a_{n-1}k_{n-3}+a_{n-2}k_{n-4}+\cdots]S-[a_nk_{n-1}+a_{n-1}k_{n-2}+a_{n-2}k_{n-3}]=L[b(x)] \qquad (20)$$

$$Y(s)=\frac{L[b(x)]}{[a_nS^n+a_{n-1}S^{n-1}+a_{n-2}S^{n-2}+\cdots a_{n-n}S^{n-n}]}+\frac{[a_nS^{n-1}+a_{n-1}S^{n-2}+a_{n-2}S^{n-3}+\cdots a_{n-(n-1)}S^{n-n}]k}{[a_nS^n+a_{n-1}S^{n-1}+a_{n-2}S^{n-2}+\cdots a_{n-n}S^{n-n}]}+\frac{[a_nS^{n-2}+a_{n-1}S^{n-3}+a_{n-2}S^{n-4}+\cdots a_{n-(n-2)}S^{n-n}]k_1}{[a_nS^n+a_{n-1}S^{n-1}+a_{n-2}S^{n-2}+\cdots a_{n-n}S^{n-n}]}+\cdots+\frac{[a_nk_{n-2}+a_{n-1}k_{n-3}+a_{n-2}k_{n-4}+\cdots]S}{[a_nS^n+a_{n-1}S^{n-1}+a_{n-2}S^{n-2}+\cdots a_{n-n}S^{n-n}]}+\frac{[a_nk_{n-1}+a_{n-1}k_{n-2}+a_{n-2}k_{n-3}]}{[a_nS^n+a_{n-1}S^{n-1}+a_{n-2}S^{n-2}+\cdots a_{n-n}S^{n-n}]} \qquad (21)$$

Algebraically solving the equation and taking the inverse Laplace transform gives the result

$$y(x)=L^{-1}[Y(s)] \qquad (22)$$

RESULTS

Example 1:
Solve $y' + y = 0, \quad y(0) = 1$

Using the Elzaki transform, we have:

$$E(y) = \frac{av^2k + vE(c)}{a + bv}$$

$$a = 1, \quad b = 1, \quad c = 0, \quad \text{and } k = 1$$

Substituting the above values, we have

$$E(y) = \frac{v^2}{1 + v}$$

By taking inverse Elzaki transform

$$y(x) = e^{-x}$$

By the Laplace transform, we have:

$$L(y) = \frac{L(c) + ak}{aS + b}$$

$$a = 1, \quad b = 1, \quad c = 0, \quad \text{and } k = 1$$

Substituting the above values, we have

$$L(y) = \frac{1}{S + 1}$$

The inverse Laplace transform of the equation gives:

$$y(x) = e^{-x}$$

Example 2:
Solve $y' + y = cos2x, \quad y(0) = 1$
By Elzaki transform:

$$E(y) = \frac{av^2k + vE(c)}{a + bv}$$

$$a = 1, \quad b = 1, \quad c = \cos 2x, \quad \text{and} \quad k = 1$$

Substituting the above values, we have

$$E(y) = \frac{4v^4 + v^3 + v^2}{4v^3 + 4v^2 + v + 1}$$

$$E(y) = \frac{4v^3}{5(1 + 4v^2)} + \frac{v^2}{5(1 + 4v^2)} + \frac{4v^2}{5(1 + v)}$$

By taking inverse Elzaki transform, result gives

$$y(x) = \frac{2}{5}\sin 2x + \frac{1}{5}\cos 2x + \frac{4}{5}e^{-x}$$

Laplace transform gives:

$$L(y) = \frac{L(c) + ak}{aS + b}$$

$$a = 1, \quad b = 1, \quad c = \cos 2x, \quad \text{and} \quad k = 1$$

Substituting the above values, we have

$$L(y) = \frac{S^2 + S + 4}{S^3 + S^2 + 4S + 4} = \frac{S}{5(S^2 + 4)} + \frac{4}{5(S^2 + 4)} + \frac{4}{5(S + 1)}$$

By taking the inverse Laplace transform, result gives

$$y(x) = \frac{2}{5}\sin 2x + \frac{1}{5}\cos 2x + \frac{4}{5}e^{-x}$$

Example 3:
Solve
$y'' + y = 0, \quad y(0) = y'(0) = 1$
Elzaki transform gives:

$$E(y) = \frac{v^2[ak + avP + bk + E(d)]}{a + bv + cv^2}$$

$a = 1, \quad b = 0, \quad c = 1, \quad d = 0, \quad k = 1$ and $P = 1$
Substituting the values, we have

$$E(y) = \frac{v^3 + v^2}{1 + v^2} = \frac{v^3}{1 + v^2} + \frac{v^2}{1 + v^2}$$

By taking inverse Elzaki transform, we get

$$y(x) = \sin x + \cos x$$

Laplace transform method gives:

$$L(y) = \frac{aSk + aP + bk + L(d)}{aS^2 + bS + c}$$

$a = 1, \quad b = 0, \quad c = 1, \quad d = 0, \quad k = 1$ and $P = 1$
By substitution of the values, we have

$$L(y) = \frac{S + 1}{S^2 + 1} = \frac{S}{S^2 + 1} + \frac{1}{S^2 + 1}$$

The inverse Laplace transform of this equation is simply obtained as:

$$y(x) = \sin x + \cos x$$

Example 4:
Solve

$$y'' + 9y = \cos 2x, \quad y(0) = 1, \quad y(\frac{\pi}{2}) = -1$$

$$E(y) = \frac{v^2[ak + avP + bk + E(d)]}{a + bv + cv^2}$$

$a = 1, \quad b = 0, \quad c = 9, \quad d = \cos 2x, \quad k = 1$ and $P = m$
Substituting the values, we have

$$E(y) = \frac{4mv^5 + 5v^4 + mv^3 + v^2}{36v^4 + 13v^2 + 1}$$
$$= \frac{v^2}{5(1+4v^2)} + \frac{mv^3}{(1+9v^2)} + \frac{4v^2}{5(1+9v^2)}$$

The inverse Elzaki transform will lead to the solution

$$y(x) = \frac{1}{5}\cos 2x + \frac{m}{3}\sin 3x + \frac{4}{5}\cos 3x$$

Since $y\left(\frac{\pi}{2}\right) = -1$, substituting $\frac{\pi}{2} = x$, then

$$m = \frac{12}{5}$$

Then,

$$y(x) = \frac{1}{5}\cos 2x + \frac{4}{5}\sin 3x + \frac{4}{5}\cos 3x$$

Laplace transform method gives the solution

$$L(y) = \frac{aSk + aP + bk + L(d)}{aS^2 + bS + c}$$

$a = 1, \quad b = 0, \quad c = 9, \quad d = \cos 2x, \qquad k = 1$ and $P = m$

Substituting the above values, we have

$$L(y) = \frac{S^3 + mS^2 + 5S + 4m}{S^4 + 13S^2 + 36}$$
$$= \frac{S}{5(S^2+4)} + \frac{4S}{5(S^2+9)} + \frac{3m}{3(S^2+9)}$$

The inverse Laplace transform of this equation is simply obtained as:

$$y(x) = \frac{1}{5}\cos 2x + \frac{m}{3}\sin 3x + \frac{4}{5}\cos 3x,$$

Since $y\left(\frac{\pi}{2}\right) = -1$, substituting $\frac{\pi}{2} = x$, gives

$m = \frac{12}{5}$, then

$$y(x) = \frac{1}{5}\cos 2x + \frac{4}{5}\sin 3x + \frac{4}{5}\cos 3x$$

Example 5:

$y'' + 4y = 9x, \qquad y(0) = 0, \qquad y'(0) = 7$

$$E(y) = \frac{v^2\left[ak + avP + bk + E(d)\right]}{a + bv + cv^2}$$

$a = 1, b = 0, c = 4, d = 9x, \ k = 0 \ and \ p = 7$

Substituting the above values, we have

$$E(y) = \frac{9v^5 + 7v^3}{1+4v^2} = \frac{9v^3}{4} + \frac{\frac{19}{8}\left(2v^3\right)}{1+4v^2}$$

Taking inverse Elzaki transform gives,

$$y(x) = \frac{9x}{4} + \frac{19}{8}\sin 2x$$

Using Laplace transform:

Laplace transform gives

$$L(y) = \frac{aSk + aP + bk + L(d)}{aS^2 + bS + c}$$

$a = 1, \quad b = 0, \quad c = 4, \quad d = 9x, \qquad k = 0$ and $P = 7$

Substituting the values, we have

$$L(y) = \frac{9 + 7S^2}{S^2\left(S^2+4\right)} = \frac{9}{4S^2} + \frac{19}{8}\left(\frac{2}{S^2+4}\right)$$

The inverse Laplace transform of this equation is simply obtained as

$$y(x) = \frac{9x}{4} + \frac{19}{8}\sin 2x$$

Example 6:

$y''' + 2y' = \cos x, \qquad y(0) = 0, \qquad y'(0) = 1, \qquad y''(0) = 0$

By the Elzaki transform

$$E(y) = \frac{v^2\left[ak + aPv + amv^2 + bkv + bPv^2 + ckv + vE(e)\right]}{a + bv + cv^2 + dv^3}$$

$a = 1, \quad b = 0, \quad c = 0, \quad d = 1 \quad e = \cos x, \qquad k = 0, \quad P = 1$ and $m = 0$

Substituting the above values, we have

$$E(y) = \frac{2v^5 + v^3}{2v^4 + 3v^2 + 1} = \frac{v^3}{1+v^2}$$

The inverse Elzaki transform will lead to the solution

$$y(x) = \sin x$$

Laplace transform gives:

$$L(y) = \frac{aS^2k + aPS + am + bkS + bP + ck + L(e)}{aS^3 + bS^2 + cS + d}$$

$a = 1, \quad b = 0, \quad c = 0, \quad d = 1 \quad e = \cos x, \qquad k = 0, \quad P = 1$ and $m = 0$

Substituting the above values, we have

$$L(y) = \frac{S^3 + 2S}{S^5 + 3S^3 + 2S} = \frac{1}{S^2 + 1}$$

The inverse Laplace transform of this equation is simply obtained as

$y(x) = \sin x$

Example 7:

Consider the ODE

$y^{iv} - 3y'' + 2y = 0$,

$y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = 0$.

By Elzaki transform

$a = 1, b = -3, c = 0, d = 0 \ e = 0, k_0 = 1, k_1 = 0, \ k_2 = 0, \ k_3 = 0$

$$E(y)\left[\frac{1 - 3v^2 + 2v^4}{v^4}\right] = \frac{1 - 3v^2}{v^2}$$

$$E(y) = \frac{v^2 - 3v^4}{1 - 3v^2 + 2v^4}$$

$$= \frac{2v^2}{1 - v^2} - \frac{v^2}{1 - 2v^2}$$

Taking inverse Elzaki gives

$y = 2 \cosh x - \cosh \sqrt{2}x$

By using the Laplace transform:

$a = 1, b = -3, c = 0, d = 0 \ e = 0, k_0 = 1, k_1 = 0, \ k_2 = 0, \ k_3 = 0$

$L(s)[s^4 - 3s^2 + 2] = s^3 - 3s$

$$L(y) = \frac{s^3 - 3s}{[s^4 - 3s^2 + 2]}$$

$$= \frac{\mathbf{2s}}{\mathbf{s^2 - 1}} - \frac{\mathbf{s}}{\mathbf{s^2 - 1}}$$

By taking inverse Laplace transform, the result gives

$y = \ cosh\ x - cosh\ \sqrt{2}\ x$

CONCLUSION

In this paper, we have demonstrated the application of the Elzaki transform and the Laplace transform in solving nth-order linear ordinary differential equation with constant coefficient and results compared with Laplace transform to show how the two methods complement each other in solving the same class of problems. The two transforms give flexibility. Elzaki transform often shorten computations and extends applicability, while Laplace transform provides familiarity and robustness.

REFERENCE

Aboodh K.S. (2013). 'The New Integral Transform "Aboodh Transform". *Global Journal of Pure and Applied Mathematics*. Volume 9. Number 1, pp.35-43.

Advanced Engineering Mathematics by H. K. Dass pp. 885 – 932.

Amurawaye F.F, Oyetoro G.G ,Odesola A.S., &Ogunfowote T.O(2022): Application of Aboodh Transforms to the solution of Nth Order Ordinary Differential Equations. *Science world Journal*, 17(4): 463 – 466.

Areo, E. A., Adeyanju N.O., & Kayode S.J.,(2020): Direct solution of second order ordinary differential equations using a class of hybrid block methods. *FUOYE Journal of Engineering and Technology*. 5(2):48-53.

Assabaai M.A., & Ahmed K.,(2023): Exact solutions for some singular Linear Ordinary Differential Equations of High Orders via NBI Polynomials. *Asian Research Journal of Mathematics*. 19(8):89-102.

Belgacem, F.B.M., & Karaballi, A. A (2006). Sumudu transform fundamental properties investigations and applications. *Journal of Applied Mathematics and Stochastic Analysis.* 3, 1-23.

Chopade P.P, & Devi S.B. (2015). 'Application of Elzaki Transform to Ordinary Differential Equation and partial Differential Equations'. *International Journal of Advance Research in Computer Science and software Engineering.* 5(3), 38 – 41.

Debnath, L., & Bhatta, D. (2007). Integral Transforms and Their Applications. *Chapman and Hall/ CRC*.

Doetsch, G. (1974). Introduction to the Theory and Application of the Laplace Transformation. Springer-verlag.

Fadugba S.E., Ogunyebi S.N., Adebayo K.J.,&OkunlolaJ.T.,(2020): Review of some Numerical Methods for Solving Initial Value Problems for Ordinary Differential Equations. *International Journal of Applied Mathematics and Theoretical Physics*. 6(1)7-13.

Shuenn-Yih Chang (2021): A family of matrix coefficient formulas for solving ordinary differential equations. *Applied Mathematics and computation*. 418: 126811.

Tarig M. Elzaki& Salih M. Elzaki (2011): "On the New Integral Transform" *Globa Journal of pure and Applied Mathematics*. .7(1), 57-64.

Tarig M. Elzaki& Salih M. Elzaki(2011): On the Elzaki transform and Ordinary Differential Equation with variable coefficients. *Advances in Theoretical and Applied Mathematics*. 6(1), 13-18.

Tarig M. Elzaki& Salih M. Elzaki(2011): On the Connections between Laplace and Elzaki transforms. *Advances in Theoretical and Applied Mathematics*. 6(1), 1-11.

William G. (2021): The Laplace transform as an alternative general method for solving linear ordinary differential equations. *STEM Education*, 1(4): 309-329.

APPENDIX

Elzaki and Laplace Transform of some important functions

Functions $F(t)$	**Elzaki Transform** $E[F(t)]$	**Laplace Transform** $L[F(t)]$
$F(1)$	v^2	$\frac{1}{S}$
$[F(t)]$	v^3	$\frac{1}{S^3}$
$F(t^n)$	$n!v^{n+1}$	$\frac{n!}{S^{n+1}}$
$F(e^{bt})$	$\frac{v^2}{1-bv}$	$\frac{1}{S-b}$
$F(\sin bt)$	$\frac{bv^3}{1+b^2v^2}$	$\frac{b}{S^2+b^2}$
$F(\cos bt)$	$\frac{v^2}{1+b^2v^2}$	$\frac{S}{S^2+b^2}$
$F[y'(t)]$	$\frac{E(y)}{v}-vy(0)$	$SL(y)-y(0)$
$F[y''(t)]$	$\frac{E(y)}{v^2}-y(0)-vy'(0)$	$S^2L(y)-Sy(0)-y'(0)$
$F[y^n(t)]$	$\frac{E(y)}{v^n}-\sum_{k=0}^{n-1}v^{2-n+k}y^k(0)$	$S^nL(y)-S^{n-1}y(0)-...-S^{n-2}y'(0)$ $-S^{n-3}y''(0)-y^{n-1}(0)$