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An advanced Taylor's Series Method to Obtain New Exact Travelling Waves Solutions of the Real-Valued Stochastic Ginzburg-Landau Equation

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Abstract

The investigations on asymptotic, semi-analytic, and numerical methods, techniques, and algorithms cannot be overemphasized or exhaustive in the field of applied sciences; while several notable equations that arise in engineering and the applied sciences in general, from several works of literature, needs to be solved and simulated to enable accurate forecasting, also study the dynamical behaviour of the system or model. In this regard, a solitary wave solution has been obtained to the real-valued stochastic Ginzburg-Landau (G-L) equation forced in the Ito sense by a multiplicative parameter in this paper by merging a very recent Zainab-Mohammed-Alwan (ZMA)) integral transform with the projected differential transform. When this noise parameter takes an arbitrary and zero value, the results via tables and graphical illustrations demonstrate remarkable convergence to the exact solution as appeared in the pieces of literature. The dynamical behaviour investigation of the system via parameter effect plots also demonstrates an increase in the concavity and superposition for each increase in the noise parameter. Inevitably, this method has been confirmed to be a perfect asymptotic alternative for solitary wave solutions through hybrid algorithms on stochastic differential equations and wider classes of differential equations (partial and ordinary), as the results proffered by this proposed method appear to be rapidly convergent compared to analytical and exact solution achieved in published literature.

Keywords: Stochastic differential equations, Ginzburg-Landau equations, ZMA transform, Modified differential transform, Convergence, Hybrid scheme

INTRODUCTION

Several models have been built over the years in prominent works of literature describing cogent phenomena and processes with the aid of differential equations. Yet, research in differential equations and methods of solution cannot be overemphasized as a general panacea to all problems encountered. Finding an accurate solution to models, whether they are linear or nonlinear, is one of the most challenging issues in computational mathematics, numerical analysis, and applied sciences. As a result, it is simpler to analyze and comprehend the dynamical behavior or pattern of models displayed (Loyinmi & Ijaola, 2024; Loyinmi & Akinfe, 2020). A differential equation usually links a function with its derivative, parameters, and under specific conditions, depending on the system or model being described. These functions serve to represent physical quantities in applications, whereas derivatives serve to indicate the rates of change, and the equation ultimately establishes the connection between these two (Loyinmi & Gbodogbe, 2024; Loyinmi & Idowu, 2023).

The evolution of computational mathematics as regards the development and erection of notable schemes, methods, and algorithms has caused a turnaround for this particular field and, in fact, made research in this field more interesting and worthwhile. Yet, it is important to still emphasize the development or invention of more schemes, as no single method is a panacea for solving all models developed. This is why it is paramount to check for the reliability and validity of some methods over others using some convergence

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checks as carried out by some authors (Loyinmi, 2024; Loyinmi & Akinfe, 2021) to confirm the validity and reliability of the hybrid technique implemented in their work (Loyinmi & Akinfe 2020b; Singh and Sharma, 2018; Agbomola & Loyinmi 2022b; Loyinmi, 2024; Necdet, 2017).

Unlike an ordinary differential equation (ODE), which is an equation composed of functions of single variables with their total derivatives, a partial differential equation (PDE) has many independent variables and a dependent variable with partial derivatives. Due to this, if we study a phenomenon that is dependent on time and a single variable, such as population dynamics through time as examined in Loyinmi, A.C., and Akinfe, T.K. (2021) (Loyinmi et al 2021; Agbomola & Loyinmi 2022a; Loyinmi & Gbodogbe 2024; Loyinmi et al 2023; Akinfe & Loyinmi 2021), a pendulum's oscillation for a predetermined amount of time, and so forth; the formulation of an appropriate model for these events uses the ordinary differential equation (linear or nonlinear). Conversely, the partial differential equation comes into play when a phenomenon, like the flow of a fluid in a channel, has multiple variables, including time. These variables include the fluid's temperature, viscosity, pressure, and the physical properties of the channel (Deniz 2013; Akinfe & Loyinmi 2022; Aziz et al, 2017; Mittal & Rajni 2016; Loyinmi & Idowu 2023; Loyinmi & Lawal, 2011). As the case may be, nature's dynamics are quite complex, and virtually all described phenomena in science and engineering have been translated into nonlinear differential equations (Loyinmi & Oredein, 2011).

As a result, partial and ordinary differential equations—especially the nonlinear equations—have drawn the attention of numerous mathematicians and applied scientists. Over the years, numerous techniques such as the Taylor collocation method, wavelet collocation method, differential quadrature method, the method of homotopy analysis (HAM), new iterative method (NIM), the method of Sumudu decomposition (SDM), the method of Elzaki decomposition (EDM), the method of perturbation iteration (PIM), variational iteration method (VIM), the method of Adomian Decomposition (ADM), the method of Elzaki homotopy transformation perturbation (EHTPM), the $\left(\frac{G'}{G}\right)$ expansion method, fractional iteration algorithm, the tanh-coth method, reproducing Kernel method, the tanh-sech method, Conjugate gradient method, Jacobi elliptic function method, the simplified bilinear method and so on, have been developed in the publications to solve nonlinear partial differential equations such as the Stochastic Ginzburg-Landau (G-L) equation, (Loyinmi & Akinfe 2020; Akinfe & Loyinmi 2021; Lot et al. 2024; Idowu & Loyinmi 2023a; Idowu & Loyinmi, 2023b; Rajarama et al. 2019; Loyinmi et al. 2017, Mehmet & Timucin 2016). It is well known that finding precise solutions to these nonlinear partial differential equations can be challenging. As a result, these equations require greater consideration and care when developing and putting into practice a suitable approach, scheme, method, or algorithm to solve them. Apart from established single asymptotic methods in the literature, some researchers have also found it appropriate to combine two effective techniques to create hybrid algorithms that improve the convergence of solutions obtained depending on the nonlinearity of the problem in question (Akinfe & Loyinmi, 2022; Idowu & Loyinmi, 2023b).

Few years back, Akinfe T.K. and Loyinmi, A.C. (2021) made a ground-breaking advancement by using their hybrid scheme (Elzaki integral transform coupled with an improved differential transform) to obtain the exact solution of the nonlinear Burgers-equation for Fisher's with all equation parameters remaining unchanged (solitary wave solutions) (Loyinmi & Lawal 2011; Loyinmi et al. 2018; Abdulghafor & Al-Rozbayani 2014, Zhaojuan & Shengfan 2015; Zainab, 2021). Additionally, convergence graphs that showed the fluid-like behavior of the equation when simulated were used to assess the validity, dependability, and authenticity of this method. This backed up the outcomes attained by employing this hybrid approach.

Loyinmi Adedapo and Akinfe Timilehin K. (2020) (Idowu & Loyinmi 2023; Abdulghafor & Al-Rozbayani 2014; Lui, 2017; Lawal et al. 2018; Lawal et al. 2017; Ning Li et al, 2015; Yasir Khan & Qingbiao, 2011; Loyinmi & Akinfe 2020a) developed an algorithm utilizing the Elzaki transform to give precise solutions to the Burgers-Huxley equation in three separate cases as a result of alterations in the equation's parameters. Furthermore, they employed a hybrid approach in (2019) that combined the homotopy perturbation method (EHTPM) with the Elzaki transform to produce accurate solutions for the Fisher family of reaction-diffusion equations (Loyinmi & Akinfe, 2020b).

Up till the 1950s, deterministic differential equation models were frequently utilized in implementations to characterize the system dynamics (Zainab 2021b; Lawal et al. 2017; Wael et al. 2021a; Wael et al. 2021b; Lawal & Loyinmi, 2012; Roger, 1988; Lawal et al, 2019, Bongsoo, 2010; Tarig, 2014; Loyinmi et al., 2017b). The phenomena that exist in the world today, however, occasionally fail to be deterministic in nature.

Furthermore, real-world stochastic disruptions arise from various unidentified sources. Noises affect statistical features and important phenomena; hence they shouldn't be ignored. Consequently, more precise mathematical representations of real events are produced in the form of stochastic differential equations.

In this paper, we shall also implement our recent novel hybrid algorithm that involves the combination of an integral transform and an improved differential transform in a more modified form as an improved version of the Elzaki transform called the ZMA transform introduced in October 2021 (Zainab, 2021b) shall be merged with the projected differential transform (PDTM), viz: the Modified Projected Differential Transform Method (MPDTM), to obtain an asymptotic solution to the real-valued stochastic Ginzburg-Landau (G-L) equation forced in the Ito sense.

In the subsequent sections, we shall illuminate the theory of the Ginzburg-Landau (G-L) equation, the real-valued G-L equation, the complex G-L equation, and the real-valued stochastic G-L equation in Section 2, and the concepts of the Weiner process (Brownian motion process), the Zainab-Mohammed-Alwan transform, and the PDTM shall be elucidated in Sections 2, and 3.

The demonstration of the suggested MPDTM on the generalized nonlinear partial differential equation, the main application of the MPDTM (Idowu & Loyinmi 2023a) to the real-valued stochastic Ginzburg-Landau equation with the multiplicative noise parameter, the results via tables with graphical illustrations and the discussion of the findings will be discussed in Sections 2.8, 2.9 to 2.11 and 3.

In conclusion, Section 8 of this research supports the investigation's result and offers a potential proposal in this field of knowledge.

MATERIAL AND METHOD

The theory of the Ginzburg-Landau Equation

The Ginzburg-Landau (G-L) equation, which bears the names of Vitaly Ginzburg and Lev Landau (2002), is a nonlinear differential equation derived from mathematical physical theory that has been employed in science and engineering to explain and model a wide range of processes.

In its initial form, the G-L equation was proposed as a phenomenological model for superconductivity, which might explain type-I superconductors without taking into account their microscopic features. Ginzburg and Landau argued that the free energy, F , of a superconductor close to a superconducting transition can be represented in relation to a complex order parameter field denoted by (1) in an attempt to establish the second-order phase transition theory Landau.

$$\xi(r)=\left|\xi(r)\right|e^{i\varpi(r)} \tag{a}$$

Despite the fact that no direct interpretation of this parameter was provided in the main paper, $\left|\xi(r)\right|^2$ is a measure of the local density here in equation (a) and $\xi(r)$ is a nonzero below a phase transition into a superconducting state, much like a quantum mechanics wave function.

The free energy, F, has a field theory form provided by equation (b) under the assumption that $|\xi(r)|$ and its gradients are small.

$$F=F_k+\alpha\left|\xi\right|^2+\frac{\beta}{2}\left|\xi\right|^4+\frac{1}{2m}\left|\left(-i\hbar\nabla-2eA\right)\xi\right|^2+\frac{\left|B\right|^2}{2\mu_0} \tag{b}$$

F_k is the free energy in the normal phase, α and β in the inceptive agreement were treated as phenomenological parameters, m is an effective mass, e is the charge of an electron,

A is the magnetic vector potential and $B = \nabla \times A$ is the magnetic field.

We can write the equation (b) concisely and arrive at the Ginzburg-Landau equation by minimizing F with respect to changes in the order parameter and the vector potential as:

$$\alpha\xi + \beta|\xi|^2 \xi + \frac{1}{2m}\left(-i\hbar\nabla - 2eA\right)^2 \xi = 0$$
$$\nabla \times B = \mu_0 j, \qquad \text{(c)}$$
$$\mathrm{j} = \frac{2e}{m}\mathrm{Re}\left\{\xi^*\left(-i\hbar\nabla - 2eA\right)\xi\right\}$$

Here in (c), j represents the dissipation-less electric current density and Re the real part. The first equation and the time-independent Schrodinger equation are alike in many ways but the nonlinear term (determines the order parameter ξ) primarily sets it apart. The second equation then provides the superconducting current.

The real-valued Ginzburg-Landau Equation

The nonlinear heat equation, also known as the real-valued Ginzburg-Landau (G-L) equation can be found in a variety of physics and chemistry contexts. The real G-L equation was first referred to as a long wave amplitude equation in the context of pattern formation in relation to convection in binary mixtures close to the onset of instability (National Center for Biotechnology and information, 2022; Wael et al., 2021b, Idowu et al 2023; Roger 1988; Lawal et al. 2019, Zhaojuan & Shengfan, 2015).

The real Ginzburg-Landau equation (RGLE) is of the form:

$$\partial_t u = N(\sigma)u \qquad u = u(x,t) \qquad (1)$$

Our focus in this research work is on the RGLE in the stochastic form forced in the Ito sense for N, a nonlinear operator that depends on some control parameter σ.

The complex Ginzburg-Landau Equation

The studies of Poiseuille flow and reaction-diffusion systems are where the complex G-L equation was first discovered. The G-L equation is complex and has the form:

$$\frac{\partial M}{\partial t} = (1+i\alpha)\frac{\partial^2 M}{\partial x^2} + M - (1+i\beta)|M|^2 M \qquad (2)$$

The solution to the equation (2) can be expressed as:

$$U = u_0 + M(x,t)e^{ik_c x + i\omega_c t} + M^*(x,t)e^{-(ik_c x + i\omega_c t)} + O(x,t) \qquad (3)$$

Here, α and β are the parameters.

It would interest you to take note that the real-valued G-L equation is simply a special case of the complex G-L equation with $\alpha = \beta = 0$ to be:

$$\frac{\partial M}{\partial t} = \frac{\partial^2 M}{\partial t^2} + M - |M|^2 M \qquad (4)$$

Also, in the limit case $\alpha, \beta \to \infty$, the complex G-L equation reduces to the Nonlinear Schrodinger equation which possesses known soliton solutions.

The real-valued stochastic Ginzburg-Landau Equation

In this research work, we are concentrating on implementing a modified differential transform algorithm coupled with an improved integral transform viz: ZMA transform on the real-valued Ginzburg-Landau equation with the multiplicative noise parameter, forced in the Ito sense coupled with a stochastic parameter β_t.

Mathematically, the real-valued Ginzburg-Landau equation as expressed in (4) is given as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2} + u - |u|^2 u \qquad (5)$$

While the stochastic real-valued G-L equation forced in the Ito sense by a multiplicative noise is given as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2} + u - |u|^2 u + \sigma\beta_t u \qquad (6)$$

Here, $\beta_t = \frac{d\beta}{dt}$ is a standard Wiener process and the equation being stochastic implies random fluctuations of the system has been considered as a stochastic equation considers random fluctuations depending on time.

The Wiener Process

The Wiener process is a continuous time stochastic process with real values that bears the name of an American mathematician by the same name who conducted research on the mathematical characteristics of Brownian motion in one dimension. Both pure mathematics and applied mathematics rely heavily on this process. Pure mathematics' investigation of continuous time martingales was sparked by the Wiener process.

It is a crucial process that allows for the description of more complex stochastic processes. As a result, while it is driving process Schramm-Loewner evolution, it also plays a crucial role in stochastic calculus, diffusion processes, and even potential theory. (10) The Wiener process can be used as a model of noise in electronics engineering because it can be used to represent the integral of a white noise Gaussian process in applied mathematics. For a standard Wiener process $\beta(t)$ on the interval $[0,T]$ that depends continuously on $t \in [0,T]$, satisfies the following:

$\beta(0)=0$

For $0 \le s < t \le T$, then,

$$\beta(t)-\beta(s) \sim \sqrt{t-s}N(0,1) \qquad (7)$$

Here in (7), $N(0,1)$ is a normal distribution with mean zero and unit variance.

For, $0 \le s < t < u < v \le T$, then, $\beta(t)-\beta(s)$ and $\beta(v)-\beta(u)$ are independent. The Wiener process with a time step dt is discretized as: $d\beta \sim \sqrt{dt}N(0,1)$

Very recently, in Wael W. Mohammed et. al (2021) using the tanh-coth method, exact solution had been obtained to the real-valued stochastic Ginzburg-Landau equation in (6) above as:

$$u(x,t)=\frac{1}{2}\begin{bmatrix}1+\sqrt{2\sigma^2+1} \\ \tanh\left[\sqrt{\frac{2\sigma^2+1}{8}}\left(x+\frac{3}{\sqrt{2}}t\right)\right]\end{bmatrix}e^{\left[\sigma\beta(t)-\sigma^2 t\right]} \qquad (8)$$

In (8), for the multiplicative noise parameter $\sigma=0$, we obtain

$$u(x,t)=\frac{1}{2}\left[1+\tanh\left(\frac{x}{2\sqrt{2}}+\frac{3}{4}t\right)\right] \qquad (9)$$

We shall study the variations in the two solutions via obtained results and graphical illustrations in the quest of this research work. From the equation (8), we can obtain the initial condition for $t=0$ as:

$$u(x,0)=\frac{1}{2}\begin{bmatrix}1+\sqrt{2\sigma^2+1} \\ \tanh\left[\sqrt{\frac{2\sigma^2+1}{8}}(x)\right]\end{bmatrix}e^{\left[\sigma\beta(0)\right]} \qquad (10)$$

And from the properties of the Wiener process, $\beta(0)=0$, and consequently, we obtain our new initial condition to be:

$$u(x,0)=\frac{1}{2}\left[1+\sqrt{2\sigma^2+1}\tanh\left[\sqrt{\frac{2\sigma^2+1}{8}}x\right]\right] \qquad (11)$$

For easy computation of our solution terms, we can re-write our initial condition in equation (11) above in exponential form as the function obtained is hyperbolic as:

$$u(x,0)=\frac{\frac{1}{2}\left(1+\sqrt{2\sigma^2+1}\right)e^{\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}+\frac{1}{2}\left(1-\sqrt{2\sigma^2+1}\right)e^{-\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}}{\left(e^{\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}+e^{-\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}\right)} \qquad (12)$$

The Zainab Mohammed Alwan (ZMA) integral transform

The well-known Laplace transform which has been utilized effectively for the solution of ordinary and partial differential equations (Zainab, 2021; Zainab, 2021b) and the Elzaki transform have been modified to create the ZMA transform. The ZMA transform integral is of the form:

$$Z_{ma}(v,s)=Z_{ma}\{\xi(t)\}=\frac{1}{s}\int_0^\infty \xi(v,t)e^{-\frac{t}{s}}dt \qquad (13)$$

Here, $\xi(t)$ is a function defined for all $t \geq 0$ and equation (13) remains valid if the integral at the right-hand side exists.

Thus, the inverse ZMA transform of the equation (13) is given as:

$$\xi(t) = Z_{ma}^{-1}\{Z_{ma}(v,s)\} \quad (14)$$

Alternatively, we can express the ZMA transform in equation (13) as:

$$Z_{ma}\{\xi(t)\} = \frac{1}{sv}\int_0^{\infty} \xi(t) e^{-\frac{t}{sv}} dt \quad (15)$$

The transform of derivatives using integration by parts gives;

$$Z_{ma}\left[\frac{\partial u}{\partial t}\right] = \frac{1}{sv} Z_{ma}(v,s) - \frac{1}{sv} u(x,0) \quad (16)$$

$$Z_{ma}\left[\frac{\partial u}{\partial x}\right] = Z'_{ma}(v,s) = \frac{dZ_{ma}(X,0)}{dx} \quad (17)$$

And subsequently, higher order derivatives are given generally by mathematical induction as:

$$Z_{ma}\left[\frac{\partial^n u}{\partial x^n}\right] = \frac{1}{s^n v^n} Z_{ma}(v,s) - \sum_{k=0}^{n-1} \frac{1}{s^{n-k} v^{n-k}} \frac{\partial^n u(x,0)}{\partial t^n} \quad (18)$$

$$Z_{ma}\left[\frac{\partial^n u}{\partial x^n}\right] = Z_{ma}^{n}(v,s) = \frac{d^n Z_{ma}(X,0)}{dx^n} \quad (19)$$

The projected differential transform

The projected differential transforms as discussed in the previous write ups (Deniz, 2013; Tarig 2014) is a modified version of the differential transform method; able to treat highly nonlinear differential equations to provide highly convergent results (Bongsoo 2010; Loyinmi et al, 2017b).

Definition: The projected differential transforms $U(X,k)$ of $u(X,t)$ with respect to the variable t at t_0 is defined by:

$$U(X,k) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(X,t)\right]_{t=t_0} \quad (20)$$

$X = (x_1, x_2, x_3, \ldots, x_n)$

where $u(X,t)$ is the function, whose solution is desired from the problem and $U(X,k)$ is the transformed function of $u(X,t)$.

The inverse transforms of $u(X,k)$ with respect to the variable t at t_0 is defined by:

$$u(X,t) = \sum_{k=0}^{\infty} U(X,k)(t-t_0)^k \quad (21)$$

Combining equations (20) and (21) gives:

$$u(X,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(X,t)\right]_{t=t_0} (t-t_0)^k \quad (22)$$

The above definitions lead to the fundamental operations of the PDTM given by theorem 4.2.

Theorem: Let $P(X,k), Q(X,k)$, and $R(X,k)$ be the projected differential transforms of the functions $p(X,t)$, $q(X,t)$, and $r(X,t)$ respectively, with $X = (x_1, x_2, x_3, \ldots, x_n)$, then

Linearity Property of PDTM

If $r(X,t) = \alpha p(X,t) + \beta q(X,t)$, then $R(X,k) = \alpha P(X,k) + \beta Q(X,k)$ with α and β as constants

PDTM of Products

If $r(X,t) = p(X,t) \cdot q(X,t)$, then

$$R(X,k) = \sum_{\phi_1=0}^{k} P(X,\phi_1) Q(X, k-\phi_1)$$

PDTM of Multiple Products

Suppose we have three or more functions to be transformed such that:

$$r(X,t) = p_1(X,t) \cdot p_2(X,t) \cdot p_3(X,t) \cdot p_4(X,t) \ldots \cdot p_n(X,t),$$

Then

$$R(X,k) = \sum_{k_{n-1}=0}^{k} \sum_{k_{n-2}=0}^{k_{n-1}} \sum_{k_{n-3}=0}^{k_{n-2}} \cdots \sum_{k_2=0}^{k_3} \sum_{k_1=0}^{k_2} P_1(X,k_1) P_2(X,k_2-k_1) \times \ldots \times P_{n-2}(X,k_{n-2}-k_{n-3}) P_{n-1}(X,k_{n-1}-k_{n-2}) P_n(X,k-k_{n-1})$$

PDTM of time derivatives

If $r(X,t)=\frac{\partial^n}{\partial t^n}p(X,t)$, then

$R(X,k)=(k+1)(k+2)\ldots(k+n)P(X,k+n)$

$=\frac{(k+n)!}{k!}P(X,k+n), n\in\{1,2,3,\ldots\}$.

PDTM of space derivative

If $r(X,t)=\frac{\partial^n}{\partial x_i^n}p(x_1,x_2,x_3,...,x_n,t)$, then

$R(X,k)=\frac{\partial^n}{\partial x_i^n}P(x_1,x_2,x_3,...,x_n,k)$,

$i\in\{1,2,...,n\}$, $n\in\{1,2,...\}$.

PDTM for the product of variables and time with indices

If $r(X,t)=x_1^{\alpha_1}x_2^{\alpha_2}x_3^{\alpha_3}\ldots x_n^{\alpha_n}t^{\alpha_m}$,

Then

$R(X,k)=x_1^{\alpha_1}x_2^{\alpha_2}x_3^{\alpha_3}\ldots x_n^{\alpha_n}\delta(k_m-\alpha_m)=$

$x_1^{\alpha_1}x_2^{\alpha_2}x_3^{\alpha_3}\ldots x_n^{\alpha_n}, k_m=\alpha_m$

0, otherwise

PDTM for the product of variables, problem function, and time

If $r(X,t)=x_1^{\alpha_1}x_2^{\alpha_2}x_3^{\alpha_3}\ldots x_n^{\alpha_n}t^{\alpha_m}u(X,t)$, then,

$R(X,k)=x_1^{\alpha_1}x_2^{\alpha_2}x_3^{\alpha_3}\ldots x_n^{\alpha_n}U(X,k-n)$.

Implementing the proposed "modified projected differential transform" (mpdtm) scheme on the generalized nonlinear partial differential equation

Let us consider a nonlinear partial differential equation of the form

$Du(x,t)+Ru(x,t)+N(x,t)=g(x,t)$,

given $u(x,0)=h(x)$ (23)

In this case, D is a linear differential operator of order 2, R is linear differential operator of order less than D, N is the general nonlinear differential operator, and, $g(x,t)$ is the source term or analytical function that controls the homogeneity of the equation (23). Taking the ZMA transform of the equation (23) gives:

$$Z_{ma}\left\{\begin{matrix}Du(x,t)+Ru(x,t)\\+N(x,t)\end{matrix}\right\}=Z_{ma}\{g(x,t)\} \quad (25)$$

From the equation (25) we have:

$$\begin{aligned}&Z_{ma}\{Du(x,t)\}=Z_{ma}\{g(x,t)\}\\&-Z_{ma}\{Ru(x,t)+Nu(x,t)\}\end{aligned} \quad (26)$$

By implementing the properties of the ZMA transform appropriately as clarified in the equations (16)-(19) and simplifying accordingly, we obtain:

$$\begin{aligned}&\frac{1}{(sv)^2}Z_{ma}(v,s)-\frac{1}{(sv)^2}\frac{\partial u(x,0)}{\partial t}\\&-\frac{1}{sv}u(x,0)=\\&Z_{ma}\{g(x,t)\}\\&-Z_{ma}\{Ru(x,t)+Nu(x,t)\}\end{aligned} \quad (27)$$

By multiplying through by $(sv)^2$ and re-arranging equation (27) appropriately, we obtain:

$$\begin{aligned}&Z_{ma}(v,s)=\frac{\partial u(x,0)}{\partial t}+sv\left[u(x,0)\right]\\&+sv\left[Z_{ma}\{g(x,t)\}\right]\\&-sv\left[Z_{ma}\{Ru(x,t)+Nu(x,t)\}\right]\end{aligned} \quad (28)$$

Let $\frac{\partial u(x,0)}{\partial t}=f(x)$ and since $u(x,0)=h(x)$, we plug it into equation (28) above and obtain the following as:

$$\begin{aligned}&Z_{ma}(v,s)=f(x)+sv\left[h(x)\right]+\\&sv\left[Z_{ma}\{g(x,t)\}\right]\\&-sv\left[Z_{ma}\{Ru(x,t)+Nu(x,t)\}\right]\end{aligned} \quad (29)$$

The inverse ZMA transform of the equation (29) is;

$$Z_{ma}^{-1}\left[Z_{ma}(v,s)\right]=Z_{ma}^{-1}\left[\begin{matrix}f(x)+sv\left[h(x)\right]\\+sv\left[Z_{ma}\{g(x,t)\}\right]\end{matrix}\right]$$

$$-Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{\begin{matrix}Ru(x,t)+\\Nu(x,t)\end{matrix}\right\}\right]\right]$$

Thus,

$$\begin{aligned}&U(x,t)=\phi(x,t)-\\&Z_{ma}^{-1}\left[sv\left[Z_{ma}\{Ru(x,t)+Nu(x,t)\}\right]\right]\end{aligned} \quad (30)$$

Here in (30), $\phi(x,t)$ represents the term arising from the source term $g(x,t)$ and the given initial conditions.

The next algorithm is for us to implement the PDTM on equation (30), taking the properties of PDTM into consideration to have:

$$U(x,k+1)=\phi(x,t)- Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{Ru(x,k)+Nu(x,k)\right\}\right]\right] \quad (31)$$

From equation (31), we iterate systematically to obtain the solution terms of the nonlinear partial differential equation of (23)

$$\therefore u_0(x,t)=\phi(x,t) \quad (32)$$

For $k=0$, equation (31) becomes

$$U(x,1)=-Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{Ru(x,0)+Nu(x,0)\right\}\right]\right] \quad (33)$$

And subsequently,

$$U(x,2)=-Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{Ru(x,1)+Nu(x,1)\right\}\right]\right] \quad (k=1) \quad (34)$$

$$U(x,3)=-Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{Ru(x,2)+Nu(x,2)\right\}\right]\right] \quad (k=2) \quad (35)$$

$$U(x,4)=-Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{Ru(x,3)+Nu(x,3)\right\}\right]\right] \quad (k=3) \quad (36)$$

$$\vdots$$

$$U(x,n+1)=-Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{Ru(x,n)+Nu(x,n)\right\}\right]\right] \quad (37)$$

Hence, the solution of the nonlinear differential equation (23) is given by:

$$U(x,t)=\sum_{k=0}^{n}u(x,k) \quad (38)$$

Equation (38) results into a multivariate Taylor series that rapidly converges to the exact solution.

The application of the modified pdtm scheme on the real-valued stochastic Ginzburg-Landau equation with multiplicative noise parameter

Consider the real-valued Ginzburg-Landau equation forced in the Ito sense by multiplicative noise as elucidated in equation (6), section 2 as:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+u-u^3+\sigma\beta_t u \quad (39)$$

Subject to the initial condition

$$u(x,0)=\frac{\frac{1}{2}\left(1+\sqrt{2\sigma^2+1}\right)e^{\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}+\frac{1}{2}\left(1-\sqrt{2\sigma^2+1}\right)e^{-\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}}{\left(e^{\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}+e^{-\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}\right)} \quad (40)$$

To ease our computation and avoid cumbersomeness, let $\sqrt{2\sigma^2+1}=m$ and $\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}=p$ which in turn miniaturizes our initial condition in (35) as:

$$u(x,0)=\frac{\frac{1}{2}(1+m)e^{px}+\frac{1}{2}(1-m)e^{-px}}{\left(e^{px}+e^{-px}\right)} \quad (41)$$

By taking the ZMA transform of the linear and nonlinear differential operators as illustrated in equation (25) -(29) we have:

$$Z_{ma}\left\{\frac{\partial u}{\partial t}\right\}=Z_{ma}\left\{\frac{\partial^2 u}{\partial x^2}+u-u^3+\sigma\beta_t u\right\} \quad (42)$$

$$\therefore Z_{ma}(v,s)=u(x,0) +sv\left[Z_{ma}\left[\frac{\partial^2 u}{\partial x^2}+u-u^3-\sigma\beta_t u\right]\right] \quad (43)$$

By taking the inverse ZMA transform of (43) and simplifying accordingly, we obtain:

$$u(x,t)=\frac{\frac{1}{2}(1+m)e^{px}+\frac{1}{2}(1-m)e^{-px}}{\left(e^{px}+e^{-px}\right)} +Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{\frac{\partial^2 u}{\partial x^2}+u-u^3-\sigma\beta_t u\right\}\right]\right] \quad (44)$$

Next scheme here is to implement the projected differential transform scheme on equation (44)

$$\therefore u(x,k+1)=\frac{\frac{1}{2}(1+m)e^{px}+\frac{1}{2}(1-m)e^{-px}}{\left(e^{px}+e^{-px}\right)}+$$

$$Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{\begin{array}{l}\frac{\partial^2 u(x,k)}{\partial x^2}+u(x,k)\\ -\sum_{r=0}^{k}\sum_{s=0}^{r}u(x,s)u(x,r-s)u(x,k-r)\\ -\sigma\beta_t u(x,k)\end{array}\right\}\right]\right]$$

(45)

Equation (45) can be expressed concisely as;

$$u(x,k+1)=\frac{\frac{1}{2}(1+m)e^{px}+\frac{1}{2}(1-m)e^{-px}}{\left(e^{px}+e^{-px}\right)}$$

$$+Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{\psi_1(k)+\psi_2(k)-\psi_3(k)-\psi_4(k)\right\}\right]\right]$$

(46)

Here in (46) above, $\psi_1(k),\ldots,\psi_4(k)$ are the projected differential transforms $\frac{\partial^2 u(x,k)}{\partial x^2}$, $u(x,k)$, $\sum_{r=0}^{k}\sum_{s=0}^{r}u(x,s)u(x,r-s)u(x,k-r)$, $\sigma\beta_t u(x,k)$ for the decomposed nonlinear terms respectively. Thus, we can have our first solution term from equation (44) as:

$$u_0(x,t)=\frac{\frac{1}{2}\left(1+\sqrt{2\sigma^2+1}\right)e^{\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}+\frac{1}{2}\left(1-\sqrt{2\sigma^2+1}\right)e^{-\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}}{\left(e^{\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}+e^{-\frac{\sqrt{2\sigma^2+1}}{2\sqrt{2}}x}\right)} \quad (47)$$

Now, at $k=0$, we obtain:

$$u(x,1)=Z_{ma}^{-1}\left[sv\left[Z_{ma}\left\{\begin{array}{l}\psi_1(0)\\ +\psi_2(0)\\ -\psi_3(0)-\psi_4(0)\end{array}\right\}\right]\right] \quad (48)$$

The zeroth PDTM term computation

From the equation (48),

$\psi_1(0)=\frac{\partial^2 u(x,0)}{\partial x^2}$; $\psi_2(0)=u(x,0)$;

$\psi_3(0)=\sum_{s=0}^{0}u(x,s)\sum_{s=0}^{r=0}u(x,r-s)\sum_{r=0}^{k=0}u(x,k-r)$; $\psi_4(0)=\sigma\beta_t u(x,0)$

Let $\chi_0=\psi_1(0)+\psi_2(0)-\psi_3(0)-\psi_4(0)$, then we can write that;

$$\chi_0=u_{0xx}+u_0-u_0^3-\sigma\beta_t u_0 \quad (49)$$

Thus, we can express our second solution term $u_1(x,t)$ as:

$$u_1(x,t)=Z_{ma}^{-1}\left[sv\left[Z_{ma}\left[\begin{array}{l}u_{0xx}+u_0\\ -u_0^3-\sigma\beta_t u_0\end{array}\right]\right]\right] \quad (50)$$

$$u_1(x,t)=Z_{ma}^{-1}\left[sv\left[Z_{ma}\left[\chi_0\right]\right]\right] \quad (51)$$

By computing the polynomial and derivative terms in χ_0 accordingly, we obtain

$$\chi_0=-\frac{1}{2}\frac{1}{\left(e^{\frac{\sqrt{2}}{4}mx}+e^{-\frac{\sqrt{2}}{4}mx}\right)^3}\left[\begin{array}{l}2\sigma\beta_t e^{\frac{3\sqrt{2}}{4}mx}\\ +\left[4(1+\sigma\beta_t)+m^2\right]e^{\frac{\sqrt{2}}{4}mx}\\ +\left[2(1+\sigma\beta_t)-m^2\right]e^{-\frac{\sqrt{2}}{4}mx}\end{array}\right]$$

(52)

Consequently,

$$u_1(x,t)=Z_{ma}^{-1}\left[sv\left[Z_{ma}\left[-\frac{1}{2}\frac{1}{\left(e^{\frac{\sqrt{2}}{4}mx}+e^{-\frac{\sqrt{2}}{4}mx}\right)^3}\left[\begin{array}{l}2\sigma\beta_t e^{\frac{3\sqrt{2}}{4}mx}\\ +\left[\begin{array}{l}4(1+\sigma\beta_t)\\ +m^2\end{array}\right]e^{\frac{\sqrt{2}}{4}mx}\\ +\left[\begin{array}{l}2(1+\sigma\beta_t)\\ -m^2\end{array}\right]e^{-\frac{\sqrt{2}}{4}mx}\end{array}\right]\right]\right]\right]$$

By simplifying the ZMA operators appropriately,

$$u_1(x,t)=$$

$$-\frac{1}{2}\frac{t}{\left(e^{\frac{\sqrt{2}}{4}mx}+e^{-\frac{\sqrt{2}}{4}mx}\right)^3}\begin{bmatrix}2\sigma\beta_t e^{\frac{3\sqrt{2}}{4}mx}\\+\left[4(1+\sigma\beta_t)+m^2\right]e^{\frac{\sqrt{2}}{4}mx}\\+\left[2(1+\sigma\beta_t)-m^2\right]e^{-\frac{\sqrt{2}}{4}mx}\end{bmatrix} \quad (53)$$

But we recall that $m=\sqrt{2\sigma^2+1}$; and $m^2=(2\sigma^2+1)$

$$\therefore u_1(x,t)=$$

$$-\frac{1}{2}\frac{t}{\left(e^{\frac{\sqrt{2}}{4}\left(\sqrt{2\sigma^2+1}\right)x}+e^{-\frac{\sqrt{2}}{4}\left(\sqrt{2\sigma^2+1}\right)x}\right)^3}\begin{bmatrix}2\sigma\beta_t e^{\frac{3\sqrt{2}}{4}\left(\sqrt{2\sigma^2+1}\right)x}\\+\begin{bmatrix}4(1+\sigma\beta_t)\\+2\sigma^2+1\end{bmatrix}e^{\frac{\sqrt{2}}{4}\left(\sqrt{2\sigma^2+1}\right)x}\\+\begin{bmatrix}2(1+\sigma\beta_t)\\-2\sigma^2+1\end{bmatrix}e^{-\frac{\sqrt{2}}{4}\left(\sqrt{2\sigma^2+1}\right)x}\end{bmatrix}$$

(54)

Equation (54) above gives us the second solution term for the G-L problem.

Next and similarly, for the third solution term $u_2(x,t)$, we have from the equation (46) when $k=1$, that:

$$u(x,2)=Z_{ma}^{-1}\left[sv\left[Z_{ma}\begin{Bmatrix}\psi_1(1)\\+\psi_2(1)\\-\psi_3(1)-\psi_4(1)\end{Bmatrix}\right]\right] \quad (55)$$

We shall compute the PDTM terms appropriately just as computed in 6.1

The first order PDTM computation

$$\psi_1(1)=\frac{\partial^2 u(x,1)}{\partial x^2};\psi_2(1)=u(x,1);$$

$$\psi_3(1)=\sum_{s=0}^{0}u(x,s)\sum_{s=0}^{r=0}u(x,r-s)\sum_{r=0}^{k=0}u(x,k-r)$$
$$+\sum_{r=0}^{1}\sum_{s=0}^{1}u(x,s)u(x,r-s)u(x,k-r)$$
$$+\sum_{r=1}^{1}\sum_{s=0}^{1}u(x,s)u(x,r-s)u(x,k-r)$$
$$+\sum_{r=1}^{1}\sum_{s=1}^{1}u(x,s)u(x,r-s)u(x,k-r)$$
;

$$\Rightarrow\psi_3(1)=u(x,0)u(x,0)u(x,0)$$
$$+u(x,0)u(x,0)u(x,1)+$$
$$u(x,0)u(x,1)u(x,0) \quad ;$$
$$+u(x,1)u(x,0)u(x,0)$$

$\psi_3(1)=u_0+3u_0^2u_1$; $\psi_4(1)=\sigma\beta_t u(x,1)$

Let $\chi_1=\psi_1(1)+\psi_2(1)-\psi_3(1)-\psi_4(1)$, then we can write that;

$$\chi_1=u_{1xx}+u_1-u_0^3-3u_0^2u_1-\sigma\beta_t u_0 \quad (56)$$

Thus, we can express our third solution term $u_2(x,t)$ as;

$$u_2(x,t)=Z_{ma}^{-1}\left[sv\left[Z_{ma}\begin{bmatrix}u_{1xx}+u_1-u_0^3\\-3u_0^2u_1-\sigma\beta_t u_0\end{bmatrix}\right]\right] \quad (57)$$

$$u_2(x,t)=Z_{ma}^{-1}\left[sv\left[Z_{ma}[\chi_1]\right]\right] \quad (58)$$

By computing the polynomial and derivative terms in χ_1 accordingly, we obtain χ_1

Consequently, $u_1(x,t)$

By simplifying the ZMA operators appropriately, we have third solution term as: $u_2(x,t)$ (59)

Equation (59) above gives us the second solution term for the G-L problem.

$U(x,t)$ (60)

Similarly, the terms $u_3(x,t), u_4(x,t)$ to desired term are computed in the same manner, and, we have the asymptotic solution to the real-valued Stochastic Ginzburg-Landau equation as:

(60)

The convergence of the MPDTM solution in (60) here to the exact solution is clarified and buttressed with a convergence plot in Fig. (5 - 6).

Additionally, just as we have established that for $\sigma=0$ in equation (9), we have for equation (60) that when $\sigma=0$, then:

$$u(x,t)=\frac{e^{\frac{\sqrt{2}}{4}x}}{e^{\frac{\sqrt{2}}{4}x}+e^{-\frac{\sqrt{2}}{4}x}}+\frac{3}{2}\frac{1}{\left(e^{\frac{\sqrt{2}}{4}x}+e^{-\frac{\sqrt{2}}{4}x}\right)^3}t-\frac{1}{8}\frac{\left(\begin{array}{l}4e^{\frac{5\sqrt{2}}{4}x}-9e^{-\frac{3\sqrt{2}}{4}x}+17e^{\frac{3\sqrt{2}}{4}x}\\+13e^{\frac{\sqrt{2}}{4}x}+9e^{-\frac{\sqrt{2}}{4}x}\end{array}\right)}{\left(e^{\frac{\sqrt{2}}{4}x}+e^{-\frac{\sqrt{2}}{4}x}\right)^5}t^2+\cdots$$

The above asymptotic solution also converges quickly to the exact solution in equation (9) which demonstrates par-excellent reliability of this technique, in proffering asymptotic solutions that is a duplicate of the exact solution to the G-L equation.

RESULTS AND DISCUSSION OF FINDINGS

This section of the current investigation has carefully focused on the authentication of our results obtained by our proposed MPDTM scheme. Here, we compare our results via tables and graphical illustration with the previously obtained from corresponding works of literature regarding the solution of the Ginzburg-Landau equations.

Very recently as earlier stated in the section 2 of this article, Wael W. Mohammed et.al. (2021) implemented the Tanh-Coth method to obtain the solitary wave solution of the real-valued Ginzburg-Landau (G-L) equation we have investigated in this paper for which the comparison of our asymptotic series results shall be compared to verify its reliability and the efficiency of the method. To achieve this, we have computed the result for the G-L equation when the multiplicative noise parameter σ takes an arbitrary value (its presence) and when $\sigma=0$ respectively for the Tables 1 and 2.

Furthermore, we have presented the exact and asymptotic results of the equation (60) by taking the Standard Wiener process parameter to be a unit step function $(\beta(t)=1)$, the multiplicative noise $\sigma=0.35$ and $\sigma=0$ at $x=1, x=2, x=3$ for each value of $t=0.1,0.2,0.3,0.4,0.5$ for Table 1 and 2 respectively.

3D, CONVERGENCE, AND PARAMETER EFFECT PLOTS

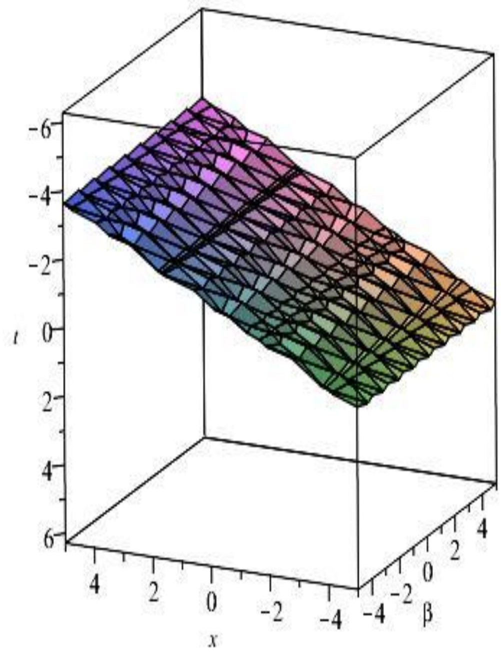

Figure 1. 3D of the exact result in equation (8) when σ takes an arbitrary value $(\sigma=0.35)$

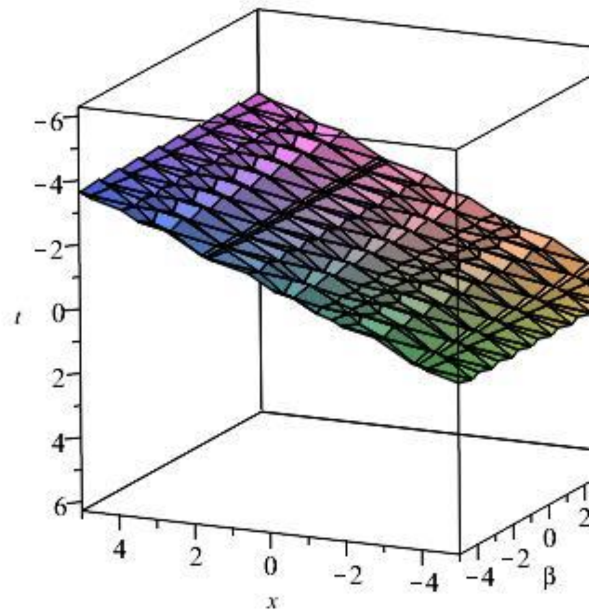

Figure 2. 3D of the asymptotic result in equation (60) when σ takes an arbitrary value $(\sigma = 0.35)$

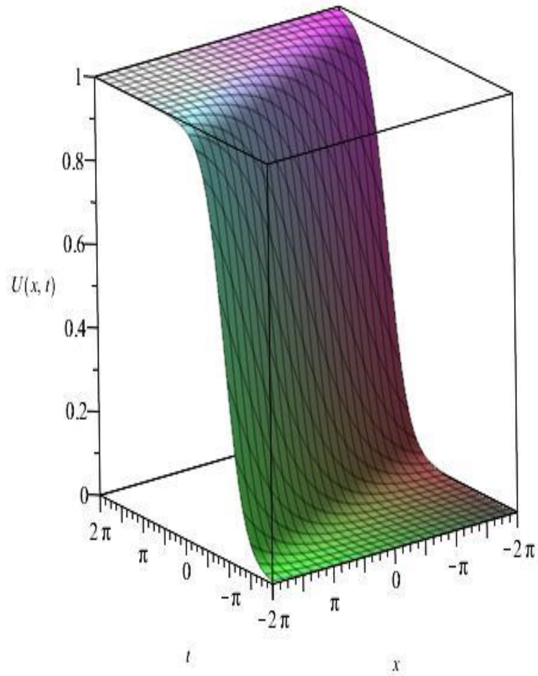

Figure 4. 3D of the asymptotic result in equation (8) when $\sigma = 0$

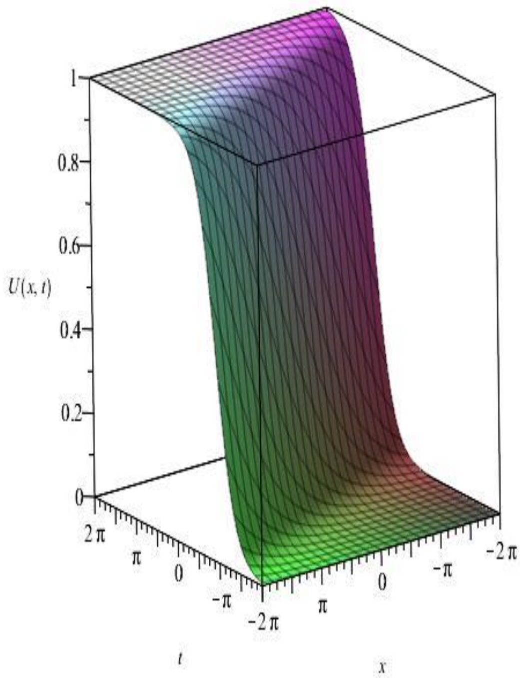

Figure 3. 3D of the asymptotic result in equation (8) when $\sigma = 0$

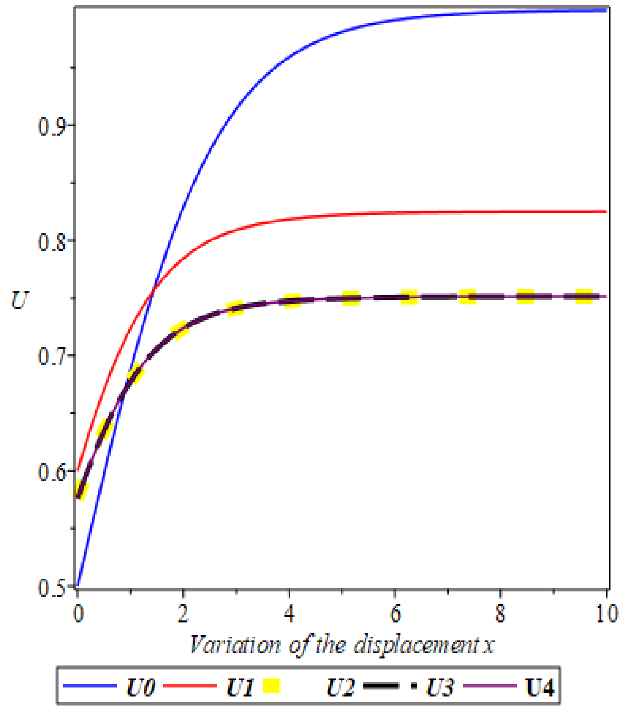

Figure 5. Convergence plot of the series solution in equation (60) when $\sigma = 0.35$

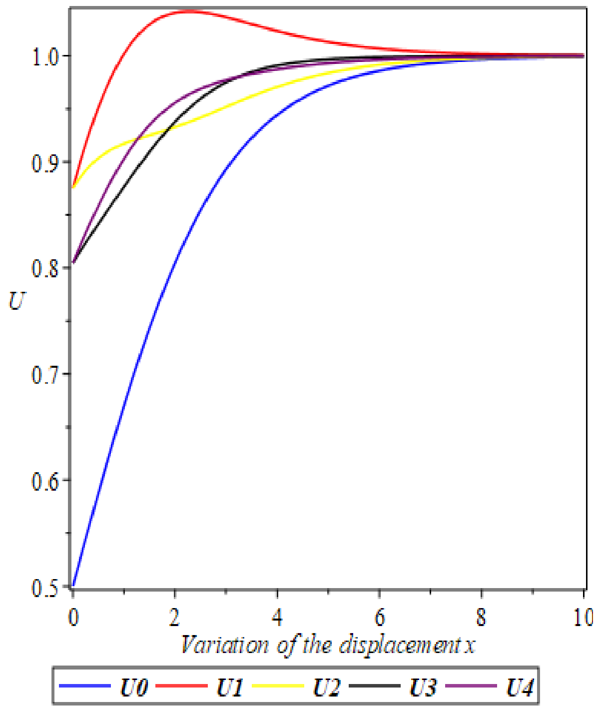

Figure 6. Convergence plot of the series solution in equation (60) when $\sigma = 0$

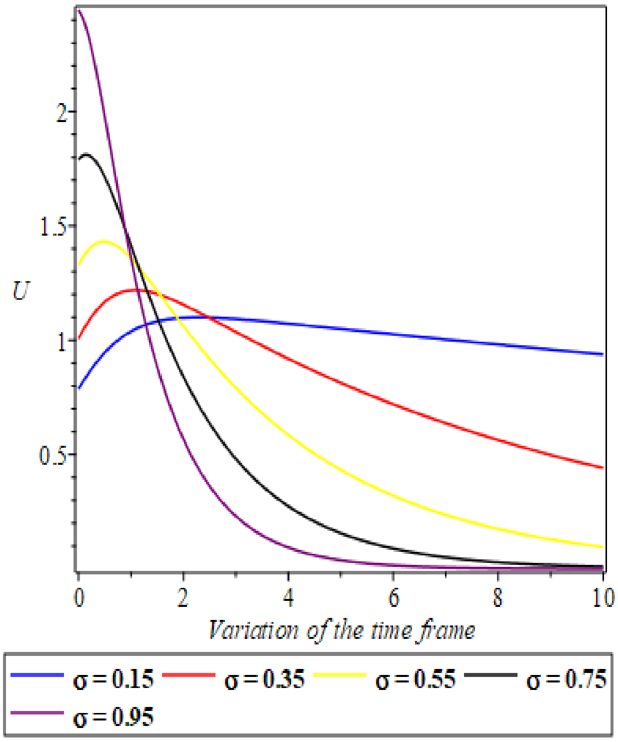

Figure 8. The impact of multiplicative noise on particle with respect to timeframe in the system

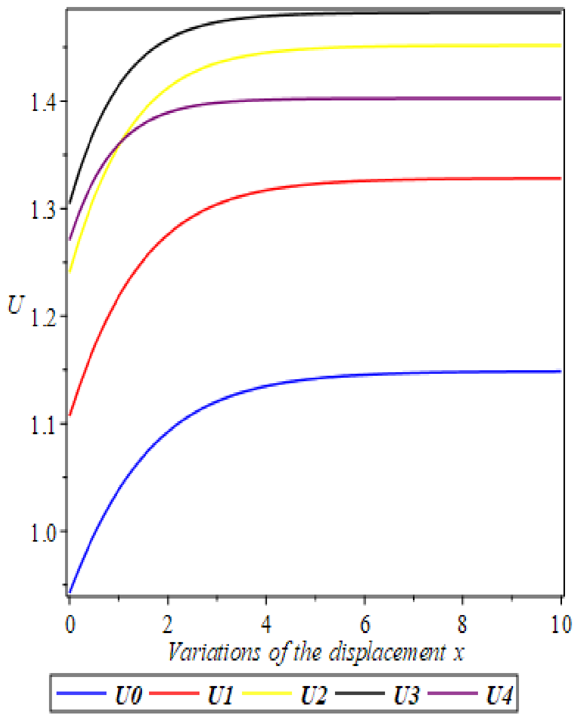

Figure 7. The impact of multiplicative noise with respect to displacement of particle in the system

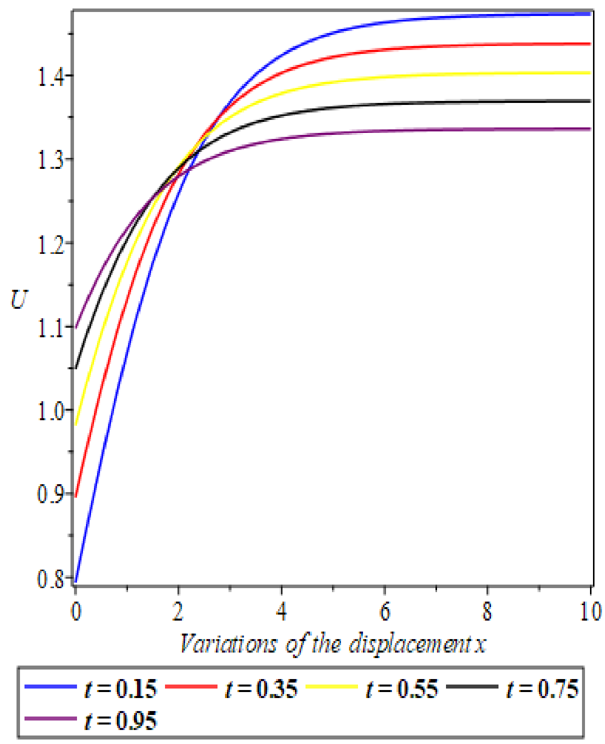

Figure 9. Impact of shifting the time interval on system particle's displacement for $\sigma = 0.35$

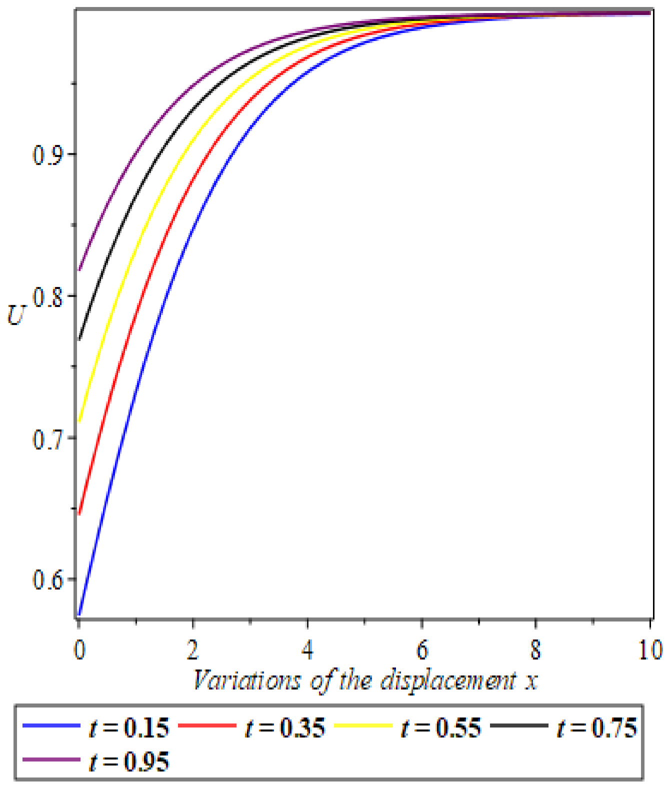

Figure 10. Impact of shifting the time interval on system particle's displacement for $\sigma = 0$

Graphical illustration remarks

The exact and asymptotic results via table of comparison and plots in three-dimensional form agreed excellently well when the multiplicative noise takes an arbitrary parameter and when it is zero. More importantly, rapid and spontaneous convergence was vividly illuminated from the convergence plots of the two cases (when $\sigma = 0.35$ and $\sigma = 0$) as the series solution of the proposed method (MPDTM) swiftly converges to the exact solution at the third iteration, whilst an increase in time and multiplicative noise parameter in turn increases the concavity of the solution function whilst exhibiting the superposition principle for a linear system.

Since the suggested Modified Projected Differential Transform Method (MPDTM) requires less computational work and is simpler than present analytical techniques in the academic literature, we can conclude with absolute certainty that it is correct, dependable, and exceedingly effective in obtaining solution for and presenting careful evaluations on stochastic differential equations and wider classes of PDEs. It also quickly converges of results in tables and plots.

Due to the method's reliability and efficiency, we hereby advocate the proposed technique (MPDTM) for the provision of exact solutions, generalized solutions, and solitary wave solutions to equations in the following fields: quantum physics, turbulence theory, dynamical systems, reaction-diffusion, fluid mechanics, stochastic dynamics, and other fields.

Table 1.

x	t	**Exact**	**MPDTM**	**Error = |Exact-MPDTM|**
$x=1$	0.1	1.04874026	1.04874025	0.00000001
	0.2	1.08585786	1.08585785	0.00000001
	0.3	1.11789584	1.11789584	0.00000000
	0.4	1.14493990	1.14493990	0.00000000
	0.5	1.16719097	1.16719087	0.00000010
$x=2$				
	0.1	1.25046805	1.25046804	0.00000001
	0.2	1.26608143	1.26608142	0.00000001
	0.3	1.27757221	1.27757218	0.00000003
	0.4	1.28535372	1.28535372	0.00000000
	0.5	1.28983980	1.28984236	0.00000256
$x=3$				
	0.1	1.36800983	1.36800983	0.00000000
	0.2	1.36775196	1.36775196	0.00000000
	0.3	1.36509375	1.36509375	0.00000000
	0.4	1.36037999	1.36038012	0.00000013
	0.5	1.35392040	1.35392192	0.00000152

Table 2.

x	t	**Exact**	**MPDTM**	**Error = |Exact-MPDTM|**
$x=1$	0.1	0.70205582	0.70205582	0.00000000
	0.2	0.73245356	0.73245356	0.00000000
	0.3	0.76404759	0.76404758	0.00000001
	0.4	0.80218388	0.80218358	0.0000003
	0.5	0.83548353	0.83547986	0.00000037
$x=2$				
	0.1	0.77294225	0.77294226	0.00000001
	0.2	0.80999843	0.80999843	0.00000000
	0.3	0.84224131	0.84224130	0.00000001
	0.4	0.86989152	0.86989128	0.00000024
	0.5	0.89330940	0.89330684	0.00000256
$x=3$				
	0.1	0.84877174	0.84877174	0.00000000
	0.2	0.87544664	0.87544664	0.00000000
	0.3	0.89798193	0.89798193	0.00000000
	0.4	0.91682730	0.91682743	0.00000013
	0.5	0.93245330	0.93245482	0.00000152

CONCLUSIONS

Since the suggested Modified Projected Differential Transform Method (MPDTM) requires less computational work and is simpler than present analytical techniques in the academic literature, we can conclude with absolute certainty that it is correct, dependable, and exceedingly effective in obtaining solution for and presenting careful evaluations on stochastic differential equations and wider classes of PDEs. It also quickly converges of results in tables and plots.

Due to the method's reliability and efficiency, we hereby advocate the proposed technique (MPDTM) for the provision of exact solutions, generalized solutions, and solitary wave solutions to equations in the following fields: quantum physics, turbulence theory, dynamical systems, reaction-diffusion, fluid mechanics, stochastic dynamics, and other fields.

REFERENCES

Abdulghafor, M.A. & Al-Rozbayani. (2014). Numerical Solution of Allen-Cahn Equation by Adomian Decomposition Method. *AL-Rafidain Journal of Computer Sciences and Mathematics*. 11(1): 69-79. https://www.iasj.net/iasj/download/cf7f9c02a5116783

Agbomola, J.O. & Loyinmi, A.C. (2022). Mathematical model for the dynamical behavior of Ebola transmission in human-bat population: implication of immediate discharge of recovered individuals. *Preprints.* https://doi.org/10.21203/rs.3.rs-1399224/v1

Agbomola, J.O. & Loyinmi, A.C. (2022). Modelling the impact of some control strategies on the transmission dynamics of Ebola virus in human-bat population: An optimal control analysis. *Heliyon*, 8:e12121. https://doi.org/10.1016/j.heliyon.2022.e12121

Akinfe, K.T. & Loyinmi, A.C. (2022). The implementation of an improved differential transform scheme on the

Deniz Sinan. (2013). Applications of Taylor collocation method and Lambert W function to the systems of delay differential equations. *Turkish Journal of Mathematics and Computer Sciences.* 20130033. 13.

Idowu, K.O. & Loyinmi, A.C. (2023). Impact of contaminated surfaces on the transmission dynamics of corona virus disease (Covid-19). *Biomed J. Sci. Tech. Res*. 51: 42280-90. https://doi.org/10.26717/BJSTR 2023.51008046

Schrodinger equation governing wave-particle duality in quantum physics and optics. *Results in physics.* https://doi.org/10.1016/j.rinp.2022.105806

Akinfe, T.K. & Loyinmi, A.C. (2021). A solitary wave solution to the generalized Burgers-Fisher's equation using an improved differential transform method: A hybrid scheme approach. *Heliyon:* 7(5):116 .https://doi.org/10.1016/j.heliyon.2021.e07001

Akinfe, T.K. & Loyinmi, A.C. (2022). An improved differential transform scheme implementation on the generalized Allen Cahn equation governing oil pollution dynamics in oceanography. *Partial Differential. Equations in Applied Mathematics..* 6: 100416. https://doi.org/10.1016/j.padiff.2022.100416

Aziz Imra, Islam Siraj-ul-Islam & Asif Muhammad. (2017). Haar wavelet collocation method for three-dimensional elliptic partial differential equations. *Computers and Mathematics with Applications.* 73. 10.1016/j.camwa.2017.02.034

Bongsoo Jang. (2010). Solving Linear and nonlinear initial value problems by the projected differential transform method. *Computer Physics Communications*. 181(5): 848-854. DOI: http://doi.org/10.1016/j.cpc.2009.12.020

.

Idowu, K.O. & Loyinmi, A.C. (2023). The analytical solution of non-linear Burgers- Huxley equations using the Tanh method. *Al-Bahir journal for Engineering and Pure Sciences.* 3(1): 68 – 77. https://doi.ord/10.55810/2312-5721.1038

Idowu, K.O., Akinwande, T.G., Fayemi, I., Adam, U.M. & Loyinmi, A.C. (2023). Laplace homotopy Perturbation method (LHPM) for solving system of N-dimensional non-linear partial differential equation. *Al- Bahir Journal for Engineering and Pure Sciences*. 3(1): 11-27.

https://doi.org.10.55810/2313-0083.1031.

Idowu, O.K., & Loyinmi, A.C. (2023). Qualitative analysis of the transmission dynamics and optimal control of covid-19. *EDUCATUM Journal of Science, Mathematics and Technology*. 10(1): 54-70. https://doi.org/10.37134/ejsmt.vol10.1.7.2023

Lawal, O.W., & Loyinmi, A.C. (2012). Magnetic and porosity effect on MHD flow of a dusty visco elastic fluid through horizontal plates with heat transfer. *J. Niger. Assoc. Math. Phys*. 21: 95- 104.

Lawal, O.W., & Loyinmi, A.C. (2019). Application of new iterative method for solving linear and nonlinear initial boundary value problems with non-local conditions. *Science World Journal*. 14: 100-104

Lawal, O.W., Loyinmi, A.C., & Arubi, D.A. (2017). Approximate solutions of higher dimensional linear and nonlinear initial boundary valued problems using new iterative method. *J Nigerian Assoc Math Phys*, 41: 35–40

Lawal, O.W., Loyinmi, A.C., & Ayeni, O.B. (2019). Laplace homotopy perturbation method for
solving coupled system of linear and nonlinear partial differential equation. *J. Math. Assoc. Niger*. 46(1): 83-91

Lawal, O.W., Loyinmi, A.C., & Erinle-Ibrahim L M. (2018). Algorithm for solving a generalized Hirota - Satsuma coupled KDV equation using homotopy perturbation transformed method. *Science World Journal*, 13: 23-28. www.scienceworldjournal.org

Lawal, O.W., Loyinmi, A.C., & Hassan, A.R. (2019). Finite difference solution for Magneto hydrodynamics thin film flow of a third grade fluid down inclined plane with ohmic heating. *J. Math. Assoc. Niger*. 46(1): 92- 97.

Lawal, O.W., Loyinmi, A.C., & Sowumi, S.O. (2017). Homotopy perturbation algorithm using Laplace transform for linear and nonlinear ordinary delayed differential equation. *J. Niger. Assoc. Math. Phys*. 41: 27- 34

Lot S, Lawal, O.W., & Loyinmi, A.C. (2024). Magnetic field's effect on two phase flow of Jeffery and non- Jeffery fluid with partial slip and heat transfer in an inclined medium. *Al – Bahir Journal for Engineering and Pure Sciences.* 4(1): 71-79. https://doi.org/10.55810/2313-0083.1054

Loyinmi, A.C., & Akinfe, T.K. (2020). An algorithm for solving the Burgers-Huxley equation using Elzaki transform, SN Applied sciences. 2: (7). https://doi.org/10.1007/s42452-019-1653-3

Loyinmi, A.C., & Akinfe, T.K. (2020). Exact solution to the family of Fisher's reaction diffusion equation using Elzaki Homotopy transformation perturbation method. *Engineering Reports.* e12084. https://doi.org/10.1002/eng2.12084

Lawal, O.W., & Loyinmi, A.C. (2012). Magnetic and porosity effect on MHD flow of a dusty visco elastic fluid through horizontal plates with heat transfer. *J. Niger. Assoc. Math. Phys*. 21: 95- 104.

Lawal, O.W., & Loyinmi, A.C. (2019). Application of new iterative method for solving linear and nonlinear initial boundary value problems with non-local conditions. *Science World Journal*. 14: 100-104

Lawal, O.W., Loyinmi, A.C., & Arubi, D.A. (2017). Approximate solutions of higher dimensional linear and nonlinear initial boundary valued problems using new iterative method. *J Nigerian Assoc Math Phys*, 41: 35–40.

Lawal, O.W., Loyinmi, A.C., & Ayeni, O.B. (2019). Laplace homotopy perturbation method for solving coupled system of linear and nonlinear partial differential equation. *J. Math. Assoc. Niger*. 46(1): 83-91.

Lawal, O.W., Loyinmi, A.C., & Erinle-Ibrahim L M. (2018). Algorithm for solving a generalized Hirota - Satsuma coupled KDV equation using homotopy perturbation transformed method. *Science World Journal,* 13: 23-28. www.scienceworldjournal.org

Lawal, O.W., Loyinmi, A.C., & Hassan, A.R. (2019). Finite difference solution for Magneto hydrodynamics thin film flow of a third-grade fluid down inclined plane with ohmic heating. *J. Math. Assoc. Niger*. 46(1): 92- 97.

Lawal, O.W., Loyinmi, A.C., & Sowumi, S.O. (2017). Homotopy perturbation algorithm using Laplace transform for linear and nonlinear ordinary delayed differential equation. *J. Niger. Assoc. Math. Phys*. 41: 27- 34.

Lot S, Lawal, O.W., & Loyinmi, A.C. (2024). Magnetic field's effect on two phase flow of Jeffery and non- Jeffery fluid with partial slip and heat transfer in an inclined medium. *Al – Bahir Journal for Engineering and Pure Sciences.* 4(1): 71-79. https://doi.org/10.55810/2313-0083.1054

Loyinmi, A.C. (2024). Analysis of two hybrid scheme to solve the Benjamin-Bona Mahony (BBM) equation. Kathmandu *University journal of science, Engineering and Technology,* 18(1), 1-15. http://dx.doi.org/10.3126/kuset.v18i1.67498

Loyinmi, A.C., & Akinfe, T.K. (2020). An algorithm for solving the Burgers-Huxley equation using Elzaki transform, *SN Applied Sciences*. 2: (7). https://doi.org/10.1007/s42452-019-1653-3

Loyinmi, A.C., & Akinfe, T.K. (2020). Exact solution to the family of Fisher's reaction diffusion equation using Elzaki Homotopy transformation perturbation method. *Engineering Reports.* e12084. https://doi.org/10.1002/eng2.12084

Loyinmi, A.C., & Gbodogbe, S.O. (2024). Mathematical modeling and control strategies for Nipah virus transmission incorporating Bat – to – pig –to – human pathway. *EDUCATUM Journal of Science, Mathematics and Technology,* 11(1): 54-80. https://doi.org/10.37134/ejsmt.vol11.1.7.2024

Loyinmi, A.C., & Idowu, K.O. (2023). Semi – analytical approach to solving Rosenau-Hyman and Korteweg-de Vries equations using integral transform. *Tanzan J Sci*, 49: 26-40. https://doi.org/10.4314/tjs.v49il.3.

Loyinmi, A.C., & Ijaola, A.L. (2024). Fractional order model of dynamical behavior and qualitative analysis of Anthrax with infected vector and saturation. *Preprints*. 2024030632. https://doi.org/10.20944/preprints202403.0632.v1

Loyinmi, A.C., & Lawal, O.W. (2011). The asymptotic solution for the steady variable-viscosity free convection flow on a porous plate. *J. Niger. Assoc. Math. Phys*.19: 273- 276.

Loyinmi, A.C., & Oredein, A.I. (2011). The unsteady variable viscosity free convection flow on a porous plate. *J. Niger. Assoc. Math. Phys*., 19: 229-232. https://www.ajol.info/index.php/jonamp/article/view/91459

Loyinmi, A.C., Akinfe, T.K., & Ojo, A.A. (2021). Qualitative analysis and dynamical behavior of a Lassa haemorrhagic fever model with exposed rodents and saturated incidence rate. *Scientific African*. e01028. https://doi.org/10.1016/j.sciaf.2021.e01028

Loyinmi, A.C., Erinle-Ibrahim, L.M., & Adeyemi, A.E. (2017). The new iterative method (NIM) for solving telegraphic equation. *J. of the Niger. Assoc. Math. Phys.,* 43: 31-36

Loyinmi, A.C., Gbodogbe, S.O., & Idowu, K.O. (2023). On the interaction of the human immune system with foreign body: mathematical modeling approach. *Kathmandu University Journal of Science, Engineering and Technology*. 17(2): 1-17. https://journals.ku.edu.np/kuset/article/view/137

Loyinmi, A.C., Lawal, O.W., & Sottin, D.O. (2017). Reduced differential Transform method for solving partial integro differential equation. *J. of the Niger. Assoc. of Math. Phys.*, 43: 37-42.

Loyinmi, A.C., Oredein, A.I., & Prince, S.U. (2018). Homotopy adomian decomposition method for solving linear and nonlinear partial differential equations. *Tasued J. Pure Appl. Sci.* 1: 254-260.

Lui SH. (2017). Legendre spectral collocation in space and time for PDEs. *Numerishe Mathematik*, 136: 75-99. https://doi.org/10.1007/s00211-016-0834-x

Mehmet Senol, Timucin Dolapci. (2016). On the Perturbation-Iteration Algorithm for fractional differential equations. *Journal of King Saud University – Science*. 28(1):69-74. https://doi.org/10.1016/j.jksus.2015.09.005

Mittal, R.C., & Rajni, R.R. (2016). A Study of One Dimensional Nonlinear Diffusion Equations by Bernstein Polynomial Based Differential Quadrature Method, *Journal of Mathematical Chemistry*. 1-23.

Necdet Bilidik. (2017). General Convergence Analysis for the Perturbation Iteration Technique, *Turk. J. Math. Comput. Sci.* 1-9.

Ning Li, Bo Meng, Xinlong Feng & Dongwei Gui. (2015). The spectral collocation method for stochastic Allen-Cahn equation via generalized polynomial chaos. *Numerical Heat Transfer, Part B: Fundamental*. 68(1): 11-29. https://doi.org/10.1080/10407790.2014.977180

Rajarama Mohan Jena, Chakraverty, S., & Subrat Kumar Jena. (2019). Dynamic response analysis of fractionally damped beams subjected to external loads using Homotopy Analysis Method (HAM). *Journal of Applied and Computational Mechanics.* 5: 355-366. 10.22055/JACM.2019.27592.1419.

Roger Temam. (1988). Infinite-Dimensional Dynamical Systems in Mechanics and Physics, Applied Mathematics Sciences. *Springer-Verlag NewYork.*1. https://doi.org/10.1007/978-1-4684-0313-8

Singh, P., & Sharma, D. (2018). Convergence and Error Analysis of Series Solution of Nonlinear Partial Differential Equation. *Nonlinear Engineering.* 7(4): 303-308. http://doi.org/10.1515/nleng-2017-0113

Tarig M. Elzaki. (2014). Application of Projected differential transform method on Nonlinear partial differential equations with proportional delay in one variable, *World Applied Sciences Journal*; 30(3): 345-349. http://doi.org/10.5829/idosi.wasj.2014.30.03.1841

The National Center for Biotechnology information advances science and health (NCBI) 2022. https://www.ncbi.nlm.nih.gov/

Wael, W.M., Hijaz Ahmad, Amjad EH, Aly ES, El-Morshedy, M., & Elabbasy, E.M. (2021). The Exact solutions of the stochastic Ginzburg-Landau equation, *Results in Physics.* 23: 103988. https://doi.org/10.1016/j.rinp.2021.103988

Wael, W.M., Raed Qahiti, Hijaz Ahmad, Jamel Baili, Fatma Elahraa Mansour & El-Morshedy,M. (2021). Exact solutions for the system of stochastic equations for the ion sound and Langmuir waves, *Results in Physics*. 30: 104841. DOI: http://doi.org/10.1016/j.rinp.2021.104841

Yasir Khan & Qingbiao, Wu. (2011). Homotopy perturbation transformation method for nonlinear equations using He's polynomials. *Computers and Mathematics with Applications*. 61: 1963-1967. https://doi.org/10.1016/j.camwa.2010.08.022

Zainab Mohammed Alwan. (2021). Solution Non Linear Partial Differential Equations By ZMA Decomposition Method, *WSEAS TRANSACTIONS on MATHEMATICS*. 2224-2880. https://doi.org/10.37394/23206.2021.20.75

Zainab Mohammed Alwan. (2021). Solution Non Linear Partial Differential Equations By ZMA Decomposition Method, *WSEAS TRANSACTIONS on MATHEMATICS*.2224-2880. https://doi.org/10.37394/23206.2021.20.75

Zainab Mohammed Alwan. (ZMA). (2021). Transform Method, Journal of Interdisciplinary Mathematics, 24(7): 1841-1849. https://doi.org/10.1080/09720502.2021.1963520

Zhaojuan Wang & Shengfan Zhou. (2015). Asymptotic Behavior of Stochastic Strongly Damped Wave Equation with Multiplicative Noise, *International Journal of Modern Nonlinear Theory and Application*. 4(3).