Analysis of Magnetohydrodynamic flow of Carreau fluid down an inclined plane with viscous and magnetic dissipation and no slip boundary condition.

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ABSTRACT

In this study, the flow of Magnetohydrodynamic (MHD) Carreau fluid down an inclined plane with viscous and magnetic dissipation and no slip boundary conditions was analyzed. The derived non-linear ordinary differential equation was solved analytically using perturbation method with MAPLE software. For no slip boundary case, the effects of some physical dimensionless parameters such as Magnetic field (*M*), Gravitational force (*G*), and Brinkmann number (*Br*) on both the velocity and temperature on the flow of the fluid were analyzed through the tables and graphs. The result shows that as Magnetic parameter increases, the velocity and temperature distribution decreases and increases as the gravitational parameter and the Brinkmann number increases with no effect on the velocity profile. For future work, Carreau and Carreau-Yasuda model can be compared using the same set of parameters.

Keywords**:** Carreau fluid, magnetohydrodynamic flow, viscous and magnetic dissipation, No slip boundary condition, MAPLE software, Newtonian and Non-Newtonian fluids.

INTRODUCTION

Liquids and gases are referred to as fluids as they flow and changes state easily due to external forces. The mechanics behind their movement is known as fluid flow. Many researchers such as Labadin and Ahmadi (2006), Lawal *et al* (2020) have worked on the influence of physical parameters on the movement of fluids through the blood vessels. Effect of mass transfer on MHD oscillatory flow for Carreau fluid which is a generalized Newtonian fluid was studied by Alkafajy and Dheia (2019), an increase in fluid movement due to the rise in temperature was observed (Tanveer *et al*, 2019). Also, stability analysis was carried out by Yahaya *et al* (2019) on MHD Carreau fluid flow over a permeable shrinking sheet with thermal radiation. Rooman *et al* (2022) investigated the Mathematical modelling of Carreau fluid flow in a renal tubule with heat transfer characteristics. The flow of power law Non-Newtonian fluid down an inclined plane considering the velocity profile using continuity and motion equations was studied by Bognar *et al* (2012), the effect of physical parameters were shown through the graphs. Tshela (2013) have studied the effect of temperature dependent variable viscosity on fluid flow down an inclined plane with a free surface.

Lawal *et al* (2020) have investigated the effects of variable viscosity on fluid flow over a convective surface. In this study, a fluid with zero velocity relative to the boundary, that is, with no slip boundary condition such as the study of Aiyesimi *et al* (2013).

In this work, the MHD flow of a Carreau fluid down an inclined plane with viscous and magnetic dissipation and no slip effect was examined. The effects of each of the physical parameters were analyzed. The governing equations were solved analytically using perturbation method (Mitga and Alkhafajy, 2019; Idrees *et al*, 2018).

MATHEMATICAL FORMULATION

We consider the flow of an incompressible non-Newtonian Carreau fluid which is assumed to be uniform and steady and moving down on an inclined plane under the influence of magnetic field. Fluids are supposed to have very small electromagnetic power with a negligible electrical conductivity.

Thinking of the system of Cartesian coordinates, the velocity vector is given as $(u(y), 0, 0)$ where u is the x-component of velocity and y is orthogonal to the x-axis

The momentum and energy equations governing the flow of the Carreau fluid are:

$$\frac{\partial \tau}{\partial y} - \sigma B_0^2 u + \rho g \sin \emptyset = 0 \quad (1)$$

with boundary conditions

$$u = 0 \text{ at } y = 0 \text{ and } \frac{\partial u}{\partial y} = 0 \text{ at } y = h \quad (2)$$

$$k \frac{\partial^2 \theta}{\partial y^2} + \tau \frac{\partial u}{\partial y} + \sigma B_0^2 u^2 = 0 \quad (3)$$

with boundary conditions

$$\theta = \theta_0 \text{ at } y = 0 \text{ and } \theta = \theta_1 \text{ at } y = h \quad (4)$$

where u is the axial velocity, θ is a fluid temperature, B_0 is a magnetic field strength, ρ is a fluid density, σ is a conductivity of the fluid, g is acceleration due to gravity and $\emptyset$ is an angle of inclination of the plane

The constitutive equation for Carreau-Yasuda fluid is given as:

$$\tau = \mu(\dot{\gamma})\dot{\gamma} = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\Gamma\dot{\gamma})^a]^{\frac{n-1}{2}} \quad (5)$$

where τ is shear stress and $\dot{\gamma}$ is the shear rate respectively, μ_0 and μ_∞ are the zero shear rate viscosity and the infinite shear rate viscosity, n is the power-law exponent and a is the Yasuda parameter model which when it is equal to 2, it makes it a Carreau parameter. "a" indicate the transition region between the zero-shear rate region and the power law region. Parameters a and n which are dimensionless determines the flow behavior of the non-Newtonian fluid with μ_0 and μ_∞.

Γ is a material time constant and;

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\pi} \quad (6)$$

where π is the second invariant strain tensor. For $n < 1$, the fluid is characterized as shear thinning fluid, shear thickening for $n > 1$ and Newtonian when $n = 1$.

Substituting equation (5) into equation (1) and (3) yield

$$\mu_0 \frac{\partial^2 u}{\partial y^2} + \frac{3}{2}\lambda^2(\mu_0 - \mu_\infty)(n-1)\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u + \rho g \sin \emptyset = 0 \quad (7)$$

$$k \frac{\partial^2 \theta}{\partial y^2} + \mu_0 \left(\frac{\partial u}{\partial y}\right)^2 + (\mu_0 - \mu_\infty)\frac{n-1}{2}\lambda^2 \left(\frac{\partial u}{\partial y}\right)^4 + \sigma B_0^2 u^2 = 0 \quad (8)$$

with thermal boundary conditions (2) and (4)

Introducing the following non-dimensionless quantities:

$\bar{y} = \frac{y}{h},\ \bar{x} = \frac{x}{h},\ \bar{u} = \frac{uh}{\mu_0},\ u = \frac{\bar{u}\mu_0}{h},\ p = \frac{\bar{p}h}{\mu_0},\ \bar{p} = \frac{p\mu_0}{h},\ \bar{T} = \frac{\theta-\theta_0}{\theta_0-\theta_1}$, equation (7) and (8) with boundary condition (2) and (4) after removing "–" becomes

$$\frac{\partial^2 u}{\partial y^2} + \frac{3}{2}We(n-1)\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y} - Mu + G = 0 \quad (13)$$

with boundary conditions

$$u = 0 \text{ at } y = 0 \text{ and } \frac{\partial u}{\partial y} = 0 \text{ at } y = 1 \quad (14)$$

$$\frac{\partial^2 T}{\partial y^2} + Br\left[\left(\frac{\partial u}{\partial y}\right)^2 + We\left(\frac{\partial u}{\partial y}\right)^4 + Mu^2\right] = 0 \quad (15)$$

with boundary conditions

$$\theta = 0 \text{ at } y = 0 \text{ and } \theta = 1 \text{ at } y = 1 \quad (16)$$

where $M = \frac{\sigma B_0^2 h}{\mu_0}$ is the Magnetic parameter, $G = \frac{\rho g \sin\emptyset\, h}{\mu_0}$ is the Gravitational Parameter $We = \frac{\lambda^2(1-\mho)}{h^3}$ is the Weissenberg number and $Br = \frac{\mu_0^3}{h^2 K(\theta_1-\theta_{0)}}$ is the Brinkman number

SOLUTION OF THE PROBLEM

Using perturbation method, let's assume the existence of small parameter $\varepsilon = We$ in equation (13) and (15). Now, we write

$$u(y) = u_0(y) + \varepsilon u_1(y) + O(We)^2 \quad (17)$$

$$T(y,\varepsilon) = T_0(y) + \varepsilon T_1(y) + O(We)^2 \quad (18)$$

Substituting equations (17) and (18) into equation (13) - (16) and collecting the like terms base on the powers of ε, gives the following equations:

Zero-order equation with boundary condition

$$\frac{\partial^2 u_0}{\partial y^2} - Mu_0 + G = 0 \quad (19)$$

$$u_o = 0 \quad \text{at} \quad y = 0 \text{ and } \frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 1 \quad (20)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + Br\left[\left(\frac{\partial u_0}{\partial y}\right)^2 + Mu_0^2\right] = 0 \quad (21)$$

$$\theta_0 = 0 \text{ at } y = 0 \text{ and } \theta_0 = 1 \text{ at } y = 1 \quad (22)$$

First-order equation with boundary condition

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{3}{2}(n-1)\left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_0}{\partial y^2} - Mu_1 = 0 \quad (23)$$

$$u_1(y) = 0 \quad \text{at} \quad y = 0 \text{ and } \frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 1 \quad (24)$$

$$\frac{\partial^2\theta_1}{\partial y^2} + Br\left[\left(\frac{\partial u_0}{\partial y}\right)^4 + 2\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y} + 2u_0u_1\right] = 0$$
(25)

$\theta_1 = 0$ at $y = 0$ and $\theta_1 = 0$ at $y = 1$
(26)

Equation (19) and (21) were solved with boundary condition (20) and (22) respectively to give

$$u_0(y) = e^{\sqrt{M}y}c_2 + e^{-\sqrt{M}y}c_1 + \frac{G}{M}$$
(27)

$$\theta_0(y) = a_7y^2 + c_5y + a_8e^{2\sqrt{M}y} + a_9e^{-2\sqrt{M}y} + c_6 - a_{10}e^{-\sqrt{M}y} - a_{11}e^{\sqrt{M}y} \quad (28)$$

Similarly, equation (23) and (25) were solved with boundary condition (24) and (26) respectively to yield $u_1(y) = \left(a_1e^{\sqrt{M}y} + \frac{a_2}{e^{\sqrt{M}y}}\right)y + a_3\left(e^{\sqrt{M}y}\right)^3 + (a_4 + c_4)e^{\sqrt{M}y} + \frac{c_3 + a_5}{e^{\sqrt{M}y}} + \frac{a_6}{\left(e^{\sqrt{M}y}\right)^3}$

(29)

$$\theta_1(y) := a_{18}y^2 + \left(-\frac{2Ga_2}{M e^{\sqrt{M}y}} - \frac{2Ga_1 e^{\sqrt{M}y}}{M} - c_2a_1\left(\left(e^{\sqrt{M}y}\right)\right)^2 - \frac{c_1a_2}{\left(e^{\sqrt{M}y}\right)^2}\right)y + a_{12}\left(\left(e^{\sqrt{M}y}\right)\right)^4 - \frac{2}{9}\frac{Ga_3\left(\left(e^{\sqrt{M}y}\right)\right)^3}{M} + \left(e^{\sqrt{M}y}\right)^2 a_{13} + \frac{a_{16}}{\left(e^{\sqrt{M}y}\right)^4} + e^{\sqrt{M}y}a_{17} + \frac{a_{15}}{\left(e^{\sqrt{M}y}\right)^2} + c_8 + \frac{a_{14}}{e^{\sqrt{M}y}} + c_7\cdot y - \frac{2}{9}\frac{Ga_6}{\left(e^{\sqrt{M}y}\right)^3 M}$$

(31)

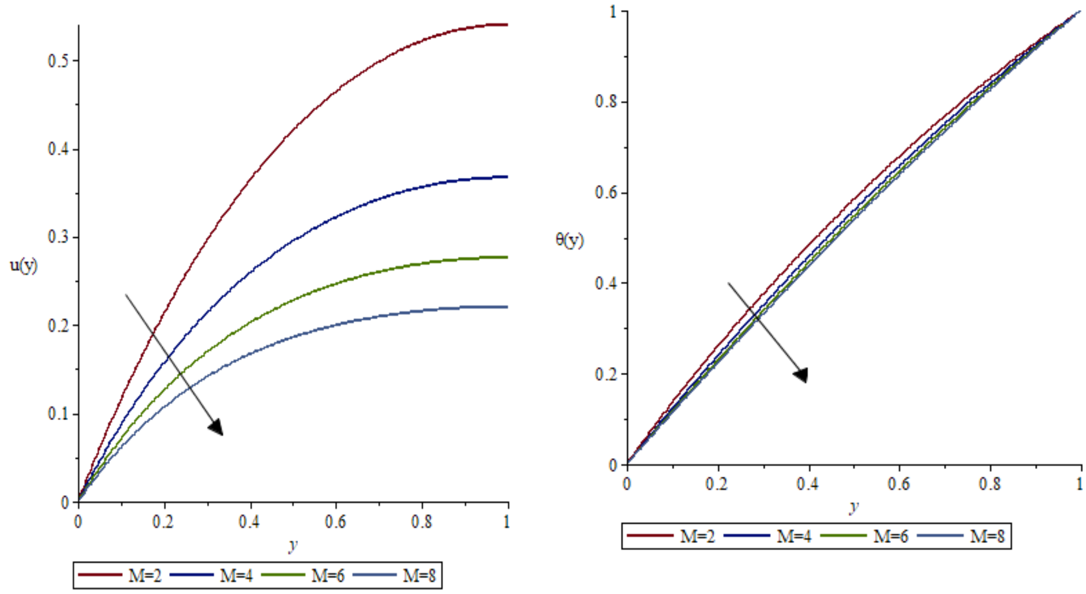
where $a_1, a_2, a_{3,\ldots\ldots\ldots\ldots,}a_{17}$ are constants.

RESULTS AND DISCUSSION

Table 1: Approximate Analytical results and Numerical results in MAPLE 20 for $M = 2$ and $M = 4$ when

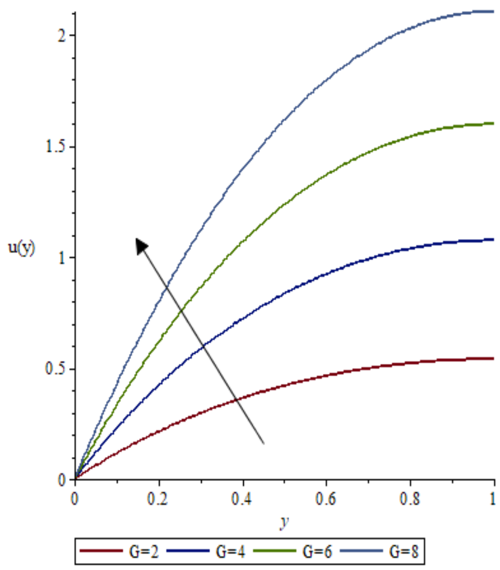
G = 2, n = 2, Br = 1 and $\varepsilon = 0.001$

$M = 2$					$M = 4$			
Approximate Analytical Results			Numerical Results using MAPLE 20		Approximate Analytical Results		Numerical Results using MAPLE 20	
y	$u(y)$	$\theta(y)$	$u(y)$	$\theta(y)$	u(y)	$\theta(y)$	u(y)	$\theta(y)$
0	0	0	0	0	0	0	0	0
0.2	0.213452	0.260124	0.214278	0.260094	0.157132	0.239071	0.157420	0.239065
0.4	0.364384	0.482603	0.365360	0.482621	0.259044	0.457042	0.259330	0.457055
0.6	0.464443	0.678447	0.465364	0.678510	0.321995	0.657478	0.322227	0.657504
0.8	0.521478	0.850806	0.522321	0.850870	0.356111	0.839072	0.356302	0.839095
1	0.539999	0.999999	0.540811	1.000000	0.366901	1.000000	0.367079	1.000000

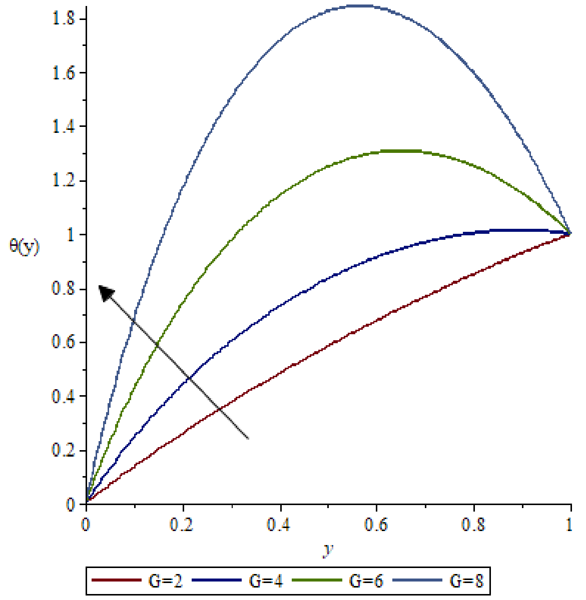


(a) (b)

Figure 1: Effects of Magnetic parameter on (a) Velocity profile (b) Temperature profile



(a)



(b)

Figure 2: Effects of Gravitational parameter on (a) Velocity profile (b) Temperature profile

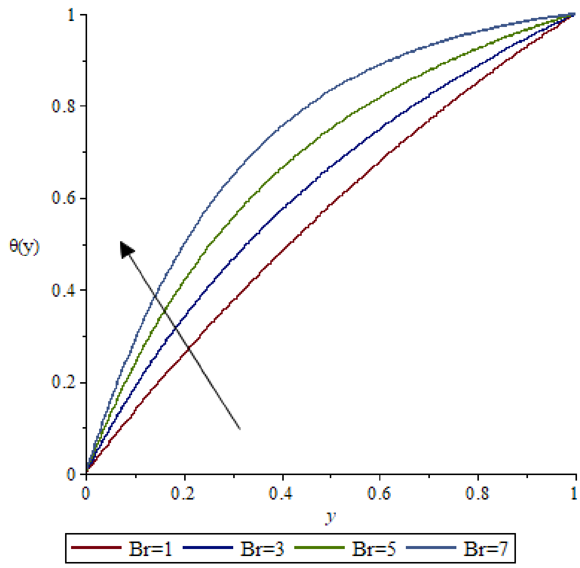


Figure 3: Effects of Brinkman number on Temperature profile

Table 1 shows the validity of the results obtained from perturbation method using numerical method in MAPLE 20. Figure 1 illustrates the effects of magnetic parameter M on the velocity and temperature distribution of the fluid when G = 2, n = 2, Br = 1 and $\varepsilon = 0.001$. it was noticed from the graphs that the velocity and temperature distribution decreases as M increases. The effects of increasing values of M is to reduce the fluid velocity and also reduce the boundary layer thickness. Then, with increase in the magnetic field parameter, the rate of transportation will be reduced when the fluid is flowing down an inclined plane.

Figure 2 illustrates the graphical illustration of both the velocity u and temperature, θ distribution for various values of G. It was observed from the graph that as the gravitational parameter G increases, the velocity and temperature distribution increases.

Figure 3 shows that the temperature distribution increases as the Brinkmann number increases while the Brinkmann number does not have effect on the velocity distribution of the fluid.

CONCLUSION

In this study, the magnetohydrodynamic flow of a thin film fluid with viscous and magnetic dissipation down an inclined plane with no slip boundary conditions were examined and analyzed. Governing equations were derived for both momentum and temperature. The effects of some physical dimensionless parameters such as Magnetic field, Gravitational force and Brinkmann number on both the velocity and temperature were observed, computed and represented graphically. The results derived from analytical method were validated numerically using finite difference method and represented in a table.

RECOMMENDATION

For future work, the effects of more or other parameters on the flow of Carreau fluid can be investigated and Carreau and Carreau-Yasuda model can be compared using the same set of parameters.

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