Published by The College of Science & Information Technology (COSIT), TASFUED, Vol. 19, No. 2, pp. 295-301.

COSIT, TASFUED
Journal of Science and Information Technology (JOSIT)

Numerical Evaluation of Fractional-Order Behaviour in a Nonlinear Climate Forecasting Model for Nigeria Temperature Anomalies Using Atangana-Baleanu Predictor-Corrector

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Abstract

Understanding long-term climate variability remains essential for environmental planning and sustainability in developing countries such as Nigeria. This study develops a nonlinear climate forecasting model with fractional-order capability for the simulation and prediction of annual temperature anomalies in Nigeria from 1960 to 2050. Annual temperature anomaly data referenced to the 1961–1990 climatological baseline were obtained from Berkeley Earth, while radiative forcing data were sourced from the Coupled Model Intercomparison Project Phase 6 (CMIP6). The model incorporates external radiative forcing, nonlinear thermal stabilization, and an Atangana–Baleanu fractional derivative framework. Numerical simulations were implemented in MATLAB R2016a using a predictor–corrector algorithm, while model performance was evaluated against Euler, Runge–Kutta fourth-order (RK4), and autoregressive AR(1) approaches using RMSE and coefficient of determination metrics. Parameter estimation yielded a = -0.023822, b = 0.060654, c = 0.072747, and $\propto$ = 1.00000. The convergence of the fractional order toward unity indicates that the calibrated annual anomaly series behaves predominantly as a classical nonlinear climate system with limited evidence of strong long-memory effects at the national aggregation scale. Sensitivity to fractional order showed $\propto$<1 unpredicted observed warming, only $\propto$ =1 tracked the trend.

Keywords: Nigeria climate model, Temperature anomaly, Nonlinear climate dynamics, Numerical simulation, CMIP6 forcing

INTRODUCTION

Climate change has emerged as one of the most pressing environmental challenges affecting both developed and developing nations. Rising global temperatures, altered precipitation patterns, desertification, and extreme weather events continue to threaten food security, water resources, biodiversity, and socioeconomic stability. In West Africa, increasing warming trends have become particularly significant because of the region’s vulnerability to climatic stress and limited adaptive infrastructure. Nigeria has experienced substantial climatic changes over recent decades, including increasing temperatures, prolonged dry seasons, heat stress, and changing rainfall variability. According to the Intergovernmental Panel on Climate Change (IPCC, 2021), warming across West Africa has accelerated since the late twentieth century due to greenhouse gas forcing and land-use changes. Similarly, the Nigerian Meteorological Agency reported persistent increases in annual temperature anomalies across several ecological zones within Nigeria (NiMet, 2023).

Mathematical climate models remain important tools for understanding long-term climatic evolution. Classical climate forecasting approaches commonly employ statistical models, regression techniques, or integer-order differential equations. However, environmental systems often exhibit delayed responses, cumulative forcing effects, and persistence across time scales. Fractional calculus has therefore gained attention because it provides a

Cite as:

Amurawaye, F. F. (2025). Numerical Evaluation of Fractional-Order Behaviour in a Nonlinear Climate Forecasting Model for Nigeria Temperature Anomalies Using Atangana-Baleanu Predictor-Corrector. *Journal of Science and Information Technology (JOSIT),* Vol. 19 No. 2, pp. 295-301.

mathematical framework capable of incorporating memory and hereditary properties into dynamical systems.

Recent studies have explored fractional differential equations in environmental and climate-related applications. Baleanu et al. (2020) demonstrated that fractional operators can improve the representation of complex dynamical behaviour in physical systems. Shah et al. (2022) further reported that fractional climate formulations may provide smoother temporal evolution under certain forcing conditions. Kumar et al. (2023) applied fractional numerical methods to environmental systems and emphasized the importance of nonlocal memory effects in long-term simulations. Ayanlade et al. (2022) observed increasing warming trends and climatic variability across West Africa, with Nigeria experiencing significant temperature escalation linked to greenhouse forcing and land-use changes. The Intergovernmental Panel on Climate Change stated in the 2021 assessment report that anthropogenic greenhouse gas emissions remain the dominant driver of global and regional warming patterns, particularly in climate-vulnerable regions of Africa. Hassan et al. (2024) identified persistent temperature variability across northern Nigeria and emphasized the need for improved regional climate forecasting frameworks capable of incorporating nonlinear environmental interactions. Abdeljawad & Alqudah (2025) noted that the application of Atangana–Baleanu fractional operators in environmental modelling provides smoother numerical behaviour and more realistic memory representation than several traditional fractional derivatives. Despite these developments, relatively limited studies have investigated nonlinear climate modelling frameworks for Nigeria using observational anomaly data and radiative forcing information. Furthermore, the present paper integrates comparative numerical simulation.

METHODOLOGY

Annual Nigeria temperature anomaly

Annual Nigeria temperature anomaly data covering 1960–2021 were obtained from Berkeley Earth. The dataset is referenced to the 1961–1990 climatological baseline and represents deviations in annual mean temperature relative to the baseline period. The use of annual observations was considered rather than monthly observations due to annual aggregation reduce short-term meteorological noise and also long-term climatic trends become more distinguishable at lower temporal resolution.

Radiative Forcing Data

Radiative forcing data were obtained from the CMIP6 forcing archive. The forcing dataset includes combined greenhouse gas and anthropogenic forcing effects commonly used in reduced-order climate simulations. The forcing series was normalized prior to numerical implementation to ensure computational stability and parameter interpretability.

The present study uses globally averaged forcing as a proxy for regional forcing because long-term Nigeria-specific forcing datasets remain limited.

Mathematical Formulation

The nonlinear climate model with fractional-order capability is expressed as:

$$^{ABC}D_t^{\alpha}T(t) = \propto T(t) + bF(t) - cT^3(t) \quad (1)$$

where:

$T(t)$ denotes annual temperature anomaly,

$F(t)$ represents normalized radiative forcing

a describes internal climate feedback

b measures forcing sensitivity

c controls nonlinear stabilization

$\propto$ represents the fractional-order parameter. $\propto \leq 1$

The nonlinear cubic term prevents unrealistic explosive warming behaviour during long-term numerical integration. The model reduces to a standard nonlinear ordinary differential equation when $\propto < 1$

The Atangana–Baleanu derivative was selected because of its non-singular kernel and smoother memory representation relative to some classical fractional operators.

Equation (1) is solved using an adaptive predictor-corrector numerical algorithm derived crom Adams-Bashforth-Moulton approached. The discretized ABC operator is expressed as

$$T_{n+1} = T_0 + \frac{1-\propto}{B(\propto)} f(T_n) + \frac{\propto h^{\propto}}{B(\propto)\,\Gamma(\alpha\}} \sum_{j=1}^{n} w_{n-j} f(T_j) \quad (2)$$

$B(\propto)$ *is the normalizing function*
w_{n-j} *denotes fractional memory weights*
h *denotes the step size*

The model parameters were calibrated using nonlinear least square optimization to minimized discrepancies between observed and simulated temperature anomalies. This is define as

$$min \sum_{i=1}^{N} \left(T_i^{obs} - T_i^{models}\right)^2 \quad (3)$$

Bootsrtap resampling was further employed to generate 95% confidence interval for fractional parameter

Model validation

Models performance were evaluated using Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Coefficient of Determination (R^2) , Akaike information criterion (AIC) and Bayesian Information Criterion (BIC). To evaluate predictive performance, the nonlinear climate model was compared against

Euler numerical simulation, RK4 numerical simulation, and an AR(1) statistical benchmark. Bootstrap resampling was additionally implemented to generate 95% confidence intervals for forecast uncertainty assessment.

The models were numerically implemented in MATLAB R2016a using a predictor–corrector integration procedure.

RESULTS AND DISCUSSION

Results

Table 1. Parameter estimation.

Parameters	Estimated Values
a	−0.023822
b	0.060654
c	0.072747
∝	1.00000

The estimated parameter values provide important physical insights into Nigeria's climate dynamics. The parameter a=-0.023822 indicates the existence of stabilizing internal climate feedback within the reduced-order system. The negative sign suggests that the climate dynamics contain damping mechanisms that resist abrupt thermal amplification. The forcing sensitivity parameter b=0.060654 confirms that increasing radiative forcing contributes positively to Nigeria's temperature anomalies. Although moderate in magnitude, the coefficient implies persistent forcing-driven warming over long time scales. The nonlinear stabilization coefficient c=0.072747 plays an important role in suppressing unrealistic runaway warming during forecast simulations. This explains the gradual stabilization behaviour observed in the long-term projections. The convergence of the fractional order ∝ toward unit (∝=1.0000) indicates that the calibrated model behaves very closely to an integer-order system. Physically, this suggests that long-memory effects in the annual Nigeria anomaly series are relatively weak at the aggregated national scale implying that Nigeria's annual anomaly evolution is dominated more strongly by external forcing and nonlinear stabilization than by persistent fractional memory effects. Consequently, the study does not claim strong evidence of long-memory climatic persistence in the aggregated annual anomaly series. This scientifically demonstrates that fractional formulations should not automatically be assumed to outperform classical systems.

Figure 1 compared the nonlinear climate model using Nigeria temperature anomaly with various values of ∝ against the observed data. At ∝ <1, the model underestimates the magnitude of warming showing no long memory. At ∝ =1, the model behaves predominately as classical nonlinear climate system rather than a fractional model and it tracks the long term rise in observed anomalies. This suggests that Nigeria national annual-mean temperature record, does not behave as if it has strong long-term memory that could smooth or delay response to improve model. It also suggests that the annual national scale temperature anomalies are dominated by the current forcing rather than by the persistence effects from decades earlier. This analysis uses one aggregate time series for Nigeria, regional or seasonal data may reveal memory effects. The result further suggests that monthly or seasonal temperature data may be more appropriate for detecting potential fractional climatic memory within Nigeria's climate system.

In Figure 2, to evaluate predictive performance of the model, the nonlinear climate model was compared against Euler numerical simulation, RK4 numerical simulation, and an AR(1) statistical benchmark. The nonlinear climate model and Euler models produced identical performance because the calibrated fractional order converged to the classical case as ∝ =1. The RK4 also got a close model fit with the nonlinear climate model. The AR(1) model under-predicts post-1990 mean temperature anomaly meaning that using purely statistical time-series models without accounting for trend can underestimate future warming in regions like west Africa where signal is non-stationary. The close match of the three models implies that Nigeria's mean temperature has been responding to forcing in a way that is well described by a first order process with little long memory effect.

In Figure 3, the forecast simulations statistically indicate that Nigeria's temperature anomalies are expected to remain persistently positive through 2050. The projections initially exhibit a slight decline after 2022 before gradually stabilizing and increasing moderately toward the later forecast years. The forecast results suggest that Nigeria may continue to experience elevated heat stress, increasing evapotranspiration, and greater climatic vulnerability, particularly within already climate-sensitive regions. The forecast from the nonlinear climate model does not predict that that Nigeria temperature will stop warming in reality. The plateau reflects the model's internal equilibrium not a climate projection because model was trained on historical data, it does not have information about future greenhouse gas emissions, aerosol changes or policy scenarios.

According to Figure 4, bootstrap-derived 95% confidence intervals reveal increasing uncertainty across the forecast horizon. The uncertainty bands remain relatively narrow during the early forecast years, indicating stable short-term predictive behaviour. However, uncertainty expands progressively toward 2050 due to accumulated forcing uncertainty, nonlinear interactions, parameter sensitivity, and structural model simplifications. This widening uncertainty pattern is consistent with long-term climate forecasting literature and highlights the importance of cautious interpretation of extended climate projections.

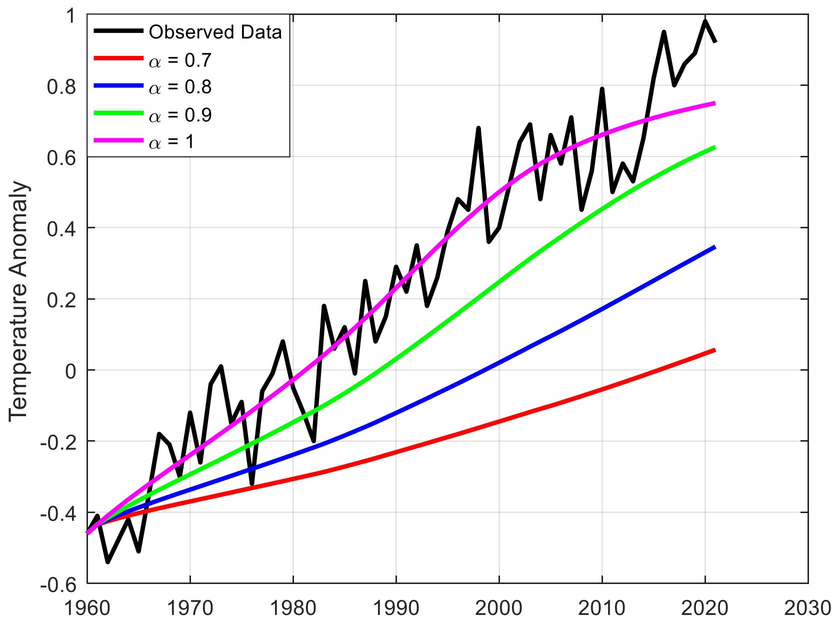

Figure 1. Fractional order simulations for Nigeria climate model.

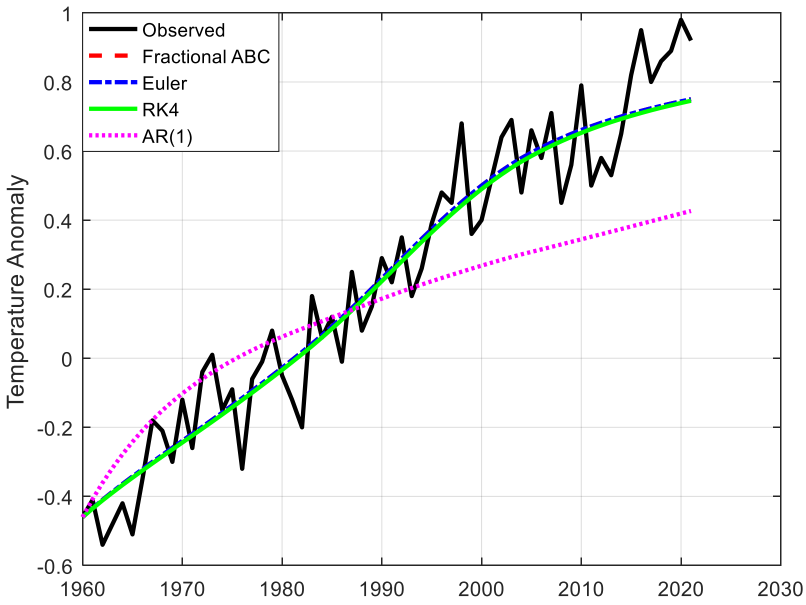

Figure 2. Nigeria climate model comparison (1960-2021).

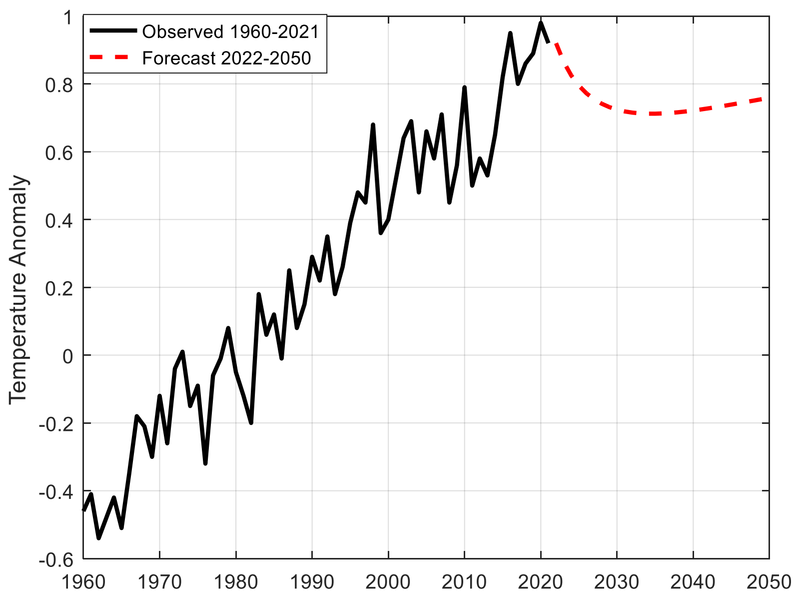

Figure 3. Nigeria temperature forecast 2022-2050.

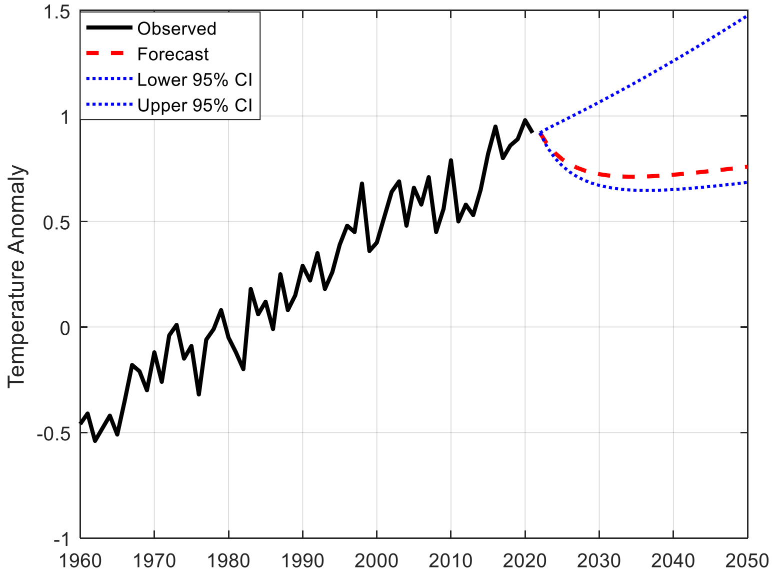

Figure 4. Nigeria climate forecast with 95% confidence interval.

Discussion

This study developed and implemented a nonlinear climate forecasting framework with fractional-order capability for Nigeria using annual temperature anomaly data from Berkeley Earth and CMIP6 forcing datasets. The results revealed persistent warming trends from 1960–2021 and projected continued positive temperature anomalies through 2050. Parameter estimation showed that the calibrated system converged toward the classical integer-order limit, indicating limited evidence of strong fractional climatic memory within the aggregated annual anomaly series. The nonlinear climate framework nevertheless outperformed the AR(1) benchmark and reproduced the long-term warming trajectory more effectively than the purely statistical approach. The study demonstrates both the usefulness and the limitations of fractional climate formulations in regional environmental modelling. More importantly, it critically evaluates the practical relevance of fractional-order modelling rather than assuming the existence of strong climatic memory a priori.

Limitations of the Study

The model dimensional does not include: atmospheric circulation processes, rainfall dynamics, ocean coupling, aerosols, or land-surface interactions.

The use of annual aggregated data may also suppress short-memory climatic fluctuations and weaken the detectability of fractional behaviour. In addition, globally averaged forcing data were used as proxies for regional forcing because long-term Nigeria-specific forcing datasets remain limited.

Future research should therefore investigate higher temporal resolution datasets, seasonal climate dynamics, machine learning comparisons, and coupled regional climate formulations..

CONCLUSION

The work demonstrates that annual aggregated anomaly data may not necessarily exhibit detectable fractional persistence after calibration but suggests that monthly or seasonal temperature data may be more appropriate for detecting potential fractional climatic memory within Nigeria’s climate system.

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