Published by The College of Science & Information Technology (COSIT), TASFUED, Vol. 20, No. 1, pp. 304-314.

COSIT, TASFUED
Journal of Science and Information Technology (JOSIT)

An Adaptive Milne Predictor-Corrector Method for Harvested Logistic Growth Model: Performance Compared with Standard Approaches

[1]Amurawaye, F. F.; [2]Soneye, R. A.; [2]Sholaja, U. O.; [1]Ogunyale, A. O.; [1]Adeogun, A. A.

[1]Department of Mathematics, Tai Solarin Federal University of Education, Ijagun Ogun State, Nigeria.
[2]Olabisi Onabanjo University, Ago-Iwoye, Ogun State, Nigeria.

Corresponding Author: amurawayeff@tasued.edu.ng
Other Authors: ogunyaleao@tasued.edu.ng; adeogunaa@tasued.edu.ng

Abstract

Population models that incorporate harvesting are essential for the sustainable management of renewable biological resources such as fisheries, forests, and wildlife populations. The harvested logistic growth model dx/dt=rx(1-x/K)-H, provides a simple yet effective framework for examining how populations respond to continuous exploitation, including conditions leading to persistence, equilibrium, or collapse. Although analytical solutions exist under ideal assumptions, practical applications often involve long simulation periods and dynamics near critical thresholds, thereby necessitating robust numerical techniques. This study develops an adaptive Milne predictor–corrector method equipped with local error estimation and automatic step-size control, and evaluates its performance against the classical fourth-order Runge–Kutta (RK4) method and the fixed-step Milne predictor–corrector scheme. The proposed method was implemented in MATLAB and tested using two biologically realistic parameter sets representing slow- and fast-growing populations under low, critical, and above-critical harvesting regimes. Numerical results show that all three methods achieved comparable levels of accuracy when validated against high-precision reference solutions. However, the adaptive Milne method consistently reduced the number of integration steps by approximately 50% in smooth regimes and by an even greater margin during population collapse, thereby improving computational efficiency. The study also reveals that the classical fixed-step Milne method may exhibit step-size dependent instability, particularly for systems with higher intrinsic growth rates, whereas the adaptive approach effectively mitigates this limitation through automatic adjustment of the step size. Furthermore, all methods predicted extinction times with errors of less than 1% relative to the reference solutions. These findings demonstrate that adaptive multistep methods provide an accurate, efficient, and numerically stable alternative for simulating harvested populations, particularly in applications involving repeated scenario analysis and decision-making near ecological thresholds.

Keywords: Harvested logistic model, Adaptive Milne method, Predictor-corrector, Runge-Kutta, Population dynamics

INTRODUCTION

Mathematical models play a fundamental role in understanding population dynamics and supporting the sustainable management of renewable biological resources. They provide a quantitative framework for analysing how populations respond to natural growth, environmental limitations, and human interventions, thereby informing decision-making in fisheries, forestry, wildlife conservation, and ecosystem management. Among these models, the logistic growth model introduced by Verhulst remains one of the most influential because it realistically incorporates environmental carrying capacity, allowing population growth to slow as available resources become limited (Verhulst, 1838;

Cite as:

Amurawaye, F. F., Soneye, R. A., Sholaja, U. O., Ogunyale, O. A., Adeogun, A. A. (2026). An Adaptive Milne Predictor-Corrector Method for Harvested Logistic Growth Model: Performance Compared with Standard Approaches. *Journal of Science and Information Technology (JOSIT)*, Vol. 20, No. 1, pp. 304-314.

Brauer, Castillo-Chavez, & Feng, 2003). Owing to its mathematical simplicity and ecological relevance, the logistic model has been widely applied to studies of microbial populations, fish stocks, forest ecosystems, and wildlife management.

In many practical situations, however, populations are subjected to continuous harvesting through activities such as fishing, hunting, logging, and agricultural exploitation. To account for these processes, a harvesting term H is incorporated into the classical logistic equation, producing the harvested logistic growth model. This extension captures the interaction between density-dependent population growth and resource extraction and has become an important mathematical tool for evaluating sustainable harvesting policies, estimating maximum sustainable yield, and identifying conditions that may lead to population persistence or extinction. The model also demonstrates the existence of critical harvesting thresholds beyond which population recovery is no longer possible, making it particularly valuable for ecological risk assessment and resource management.

Although analytical solutions of the harvested logistic model can be obtained under restrictive assumptions, practical ecological systems are often considerably more complex. Time-dependent harvesting strategies, parameter uncertainty, environmental variability, and long simulation periods frequently render analytical solutions impractical. Consequently, numerical methods have become indispensable for solving harvested population models accurately and efficiently (Hairer, Nørsett, & Wanner, 2021; Iserles, 2008). The reliability of ecological predictions therefore depends not only on the mathematical model itself but also on the efficiency, accuracy, and stability of the numerical integration technique employed.

Among the available numerical methods, the classical fourth-order Runge-Kutta (RK4) method remains one of the most widely used because of its high accuracy, robustness, and straightforward implementation. Nevertheless, RK4 is a one-step method that requires multiple function evaluations at every integration step. Consequently, long-term simulations or repeated scenario analyses often involve substantial computational cost, particularly when small step sizes are required to maintain numerical accuracy.

Linear multistep methods provide an attractive alternative by exploiting information from previously computed solution points. One of the most widely recognised approaches is the Milne predictor-corrector method, which combines an explicit prediction stage with an implicit correction stage to achieve fourth-order accuracy while reducing the number of function evaluations compared with one-step methods. Despite these computational advantages, the classical Milne method employs a fixed step size and may suffer from weak numerical instability when the selected step size is not well matched to the underlying dynamics of the problem (Shampine & Gladwell, 1995; Butcher, 2016). Such limitations become more pronounced in ecological systems exhibiting rapid transient behaviour, critical harvesting thresholds, or population collapse.

Adaptive step-size strategies offer an effective solution by automatically adjusting the integration step according to the estimated local truncation error. This allows larger step sizes during slowly varying solution intervals while reducing the step size when rapid changes occur, thereby improving both computational efficiency and numerical stability. In predictor-corrector algorithms, the difference between the predicted and corrected solutions provides a convenient estimate of the local error, making adaptive implementations computationally attractive (Butcher, 2016). Recent studies have further demonstrated that adaptive multistep methods can significantly improve numerical performance by allocating computational effort only where it is needed (Arévalo et al., 2020; Labanda, Behnoudfar, & Calo, 2021).

Recent developments in population modelling further highlight the continuing importance of efficient numerical algorithms. Zhang and Zhu (2025) investigated logistic population dynamics with harvesting pulses on a growing spatial domain and demonstrated that harvesting intensity, habitat expansion, and environmental conditions jointly determine long-term population persistence. Their findings illustrate that increasing model complexity inevitably increases computational demands, reinforcing the need for numerical methods that remain accurate, stable, and computationally efficient under diverse ecological conditions. Despite these advances, most numerical studies of harvested logistic models continue to rely primarily on RK4 or other fixed-step methods, while comparatively little attention has been devoted to adaptive Milne predictor-corrector schemes. Furthermore, systematic comparisons of their accuracy, computational efficiency, and stability across different ecological regimes remain limited.

This study addresses these gaps by developing an adaptive Milne predictor-corrector method incorporating local error estimation and automatic step-size control for the harvested logistic growth model. The proposed method is evaluated against the classical fourth-order Runge-Kutta method and the conventional fixed-step Milne predictor-corrector scheme using two biologically realistic parameter sets under low, critical, and above-critical harvesting regimes. The objectives of the study are to develop a robust adaptive integration algorithm, evaluate its numerical accuracy, computational efficiency, and stability, and assess its capability to predict equilibrium behaviour and extinction dynamics under different harvesting conditions. The principal contribution of this work is the demonstration that adaptive step-size control substantially improves computational efficiency while preserving numerical accuracy and mitigating the step-size-dependent instability associated with the classical Milne method. These findings provide a practical and reliable numerical framework for ecological simulation and resource management applications that require repeated long-term scenario analysis.

METHODOLOGY

Model Formulation and Equilibrium Analysis

The harvested logistic growth model extends the classical logistic equation by incorporating a constant harvesting term to represent the continuous removal of individuals from a population. This modification provides a more realistic representation of exploited biological populations and has been widely used in fisheries management, forestry, wildlife conservation, and renewable resource planning. The model captures the interaction between density-dependent population growth and harvesting pressure, allowing the identification of conditions under which a population persists, reaches equilibrium, or becomes extinct.

The harvested logistic model is:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - H, \qquad x(0) = x \tag{1}$$

where $x(t)$ denotes the population size at time t, $r > 0$ is the intrinsic population growth rate, $K > 0$ represents the environmental carrying capacity, and $H > 0$ denotes the constant harvesting rate.

The equilibrium points of the model satisfy

$$\frac{dx}{dt} = 0 \tag{2}$$

which gives

$$rx\left(1 - \frac{x}{K}\right) - H = 0 \tag{3}$$

After rearrangement, it yields the quadratic:

$$x^2 - Kx + \frac{Hk}{r} = 0 \tag{4}$$

The solutions of (4) is (5) and it correspond to the equilibrium points.

$$x_\pm = \frac{K \pm \sqrt{K^2 - \frac{4HK}{r}}}{2} \tag{5}$$

x_+ is the upper equilibrium and x_- is the lower equilibrium. Real equilibrium points exist when $K^2 - \frac{4HK}{r} \geq 0$. This condition implies that $H \leq \frac{rK}{4}$. Positive equilibrium exist only if $H \leq H_c$, where $H_c = \frac{rK}{4}$. At critical harvesting rate, $H = H_c$. The two equilibriums coalesce at $x^* = \frac{K}{2}$, corresponding to maximum sustainable yield. For $H > H_c$, no positive equilibrium exists and the population declines to extinction. The stability of each equilibrium points is assessed by $f'(x) = r - \frac{2rx}{K}$, from (1) with $f'(x_+) < 0$ indicating a stable equilibrium at x_+ and $f'(x_-) > 0$ indicating unstable equilibrium at x_-. A population initialized below x_- will declined towards extinction rather than recovering (Haires, 2022).

Numerical Methods

Three numerical integration methods were implemented in MATLAB R2024a to solve the harvested logistic model and compare their numerical performance:

Classical Fourth-Order Runge–Kutta Method (RK4)

The classical fourth-order Runge–Kutta method was adopted as the reference fixed-step numerical scheme because of its well-established accuracy and robustness for solving ordinary differential equations. The numerical formulation is given by

$$k_1 = f(t_{n,}x_n),\ \ k_2 = f\left(t_n + \frac{h}{2},\ x_n + \frac{hk_1}{2}\right), \dots \dots \dots \quad (6)$$

where the intermediate stage values are computed according to the standard RK4 algorithm.

Fixed-Step Milne Predictor–Corrector Method

The Milne predictor–corrector method is a fourth-order linear multistep technique consisting of an explicit prediction stage followed by an implicit correction stage.
The predictor equation is given by

$$x_{n+1}^{p} = x_{n-3} + \frac{4h}{3}(2f_n - f_{n-1} + 2f_{n-2}) \quad (7)$$

while the corrector is

$$x_{n+1}^{c} = x_{n-1} + \frac{h}{3}\left(f_{n-1} + 4f_n + f\left(t_{n+1}, x_{n+1}^{p}\right)\right) \quad (8)$$

Because Milne's method is not self-starting, the first three solution values required by the algorithm were generated using the RK4 method before commencing the predictor–corrector iterations.

Adaptive Milne Predictor–Corrector Method

To improve computational efficiency while maintaining numerical accuracy, an adaptive version of the Milne predictor–corrector method was developed. The adaptive algorithm employs the same predictor and corrector formulas as the fixed-step method but incorporates automatic step-size control based on local error estimation.

The local truncation error is estimated from the difference between the predicted and corrected solutions,

Local error estimate: $E_{n+1} = \frac{|x_{n+1}^{c} - x_{n+1}^{p}|}{\max(1, |x_{n+1}^{c}|)}$

(10)

and the subsequent step size is updated using Step-size update:

$$h_{new} = h_t . min\left[\beta\left(\frac{TOL}{E_{n+1}}\right)^{1/4}, h_{max}\right] \quad (11)$$

where the prescribed tolerance controls the desired level of numerical accuracy. If the estimated local error exceeds the specified tolerance, the current step is rejected and repeated using a smaller step size. Conversely, when the estimated error satisfies the prescribed tolerance, the corrected solution is accepted and the algorithm proceeds to the next integration step. This adaptive strategy enables the solver to employ larger step sizes during slowly varying solution intervals while automatically refining the step size whenever rapid changes occur.

Numerical Experiment

The performance of the three numerical methods was evaluated using two biologically realistic parameter sets representing populations with different intrinsic growth characteristics. For each parameter set, simulations were conducted under three harvesting regimes corresponding to low harvesting, critical harvesting, and harvesting above the critical threshold.

Table 1. Parameters Descriptions and Values.

Parameters	Description	Parameter Set 1	Parameter Set II
$\boldsymbol{r}$	intrinsic growth rate	0.50	1.2
$\boldsymbol{K}$	Carrying capacity	1000	5000
$\boldsymbol{x_0}$	Initial population	900	4500
$\boldsymbol{H_c}$	Critical harvesting rate	125	1500
$\boldsymbol{H < H_c}$	Low harvesting	50	600
$\boldsymbol{H = H_c}$	Critical harvesting	125	1500
$\boldsymbol{H > H_c}$	Above critical harvesting	150	1800

The first parameter set was selected because it provides easily interpretable equilibrium values and a clearly defined critical harvesting threshold, making it suitable for validating the theoretical behaviour of the harvested logistic model. The second parameter set represents a faster-growing population with a substantially larger carrying capacity and was chosen to assess whether the comparative performance of the numerical methods remains consistent under more demanding dynamic conditions.

For each parameter set, three harvesting scenarios were investigated: harvesting below the critical threshold to examine convergence towards a stable equilibrium, harvesting at the critical threshold to evaluate behaviour near the bifurcation point, and harvesting above the critical threshold to investigate population collapse and extinction. The initial population was selected above the unstable equilibrium to ensure that the observed dynamics resulted solely from differences in harvesting intensity.

A sufficiently long simulation interval was adopted to allow each harvesting scenario to approach its long-term behaviour while maintaining a reasonable computational cost for the fixed-step methods.

Performance Evaluation

The numerical methods were compared using three performance criteria.

1. Accuracy was assessed by comparing the numerical solutions with a high-precision reference solution generated using RK4 with a sufficiently small step size.
2. Computational efficiency was evaluated using the total number of integration steps required to complete each simulation.
3. Extinction time was defined as the first simulation time at which the population satisfied the prescribed extinction criterion. This measure provides a practical assessment of each numerical method's ability to accurately capture collapse dynamics under excessive harvesting.

RESULTS AND DISCUSSION

Results

The numerical performance of the adaptive Milne predictor–corrector method was evaluated by comparing it with the classical fourth-order Runge–Kutta (RK4) method and the conventional fixed-step Milne predictor–corrector method. The comparison focused on three important performance criteria: numerical accuracy, computational efficiency, and stability under different harvesting conditions. Two parameter sets representing distinct ecological growth characteristics were considered, with each parameter set examined under low, critical, and above-critical harvesting regimes.

Results for Parameter Set I

The first parameter set was selected to demonstrate the behaviour of the numerical methods under easily interpretable ecological conditions. The simulations examine how each numerical method performs as the harvesting rate changes from a sustainable level to the critical threshold and finally to an unsustainable harvesting regime leading to population collapse.

Low Harvesting Regime (H = 50)

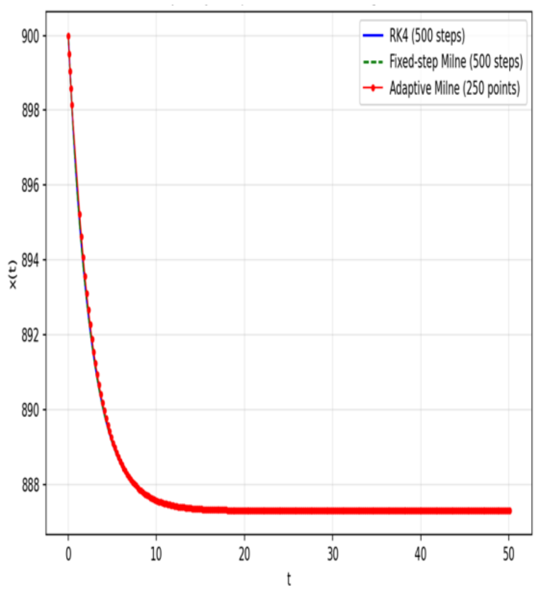

Figure 1. Compares the population trajectories obtained using RK4, the fixed-step Milne method, and the proposed adaptive Milne method under low harvesting conditions.

Critical harvesting Regime (H = 125)

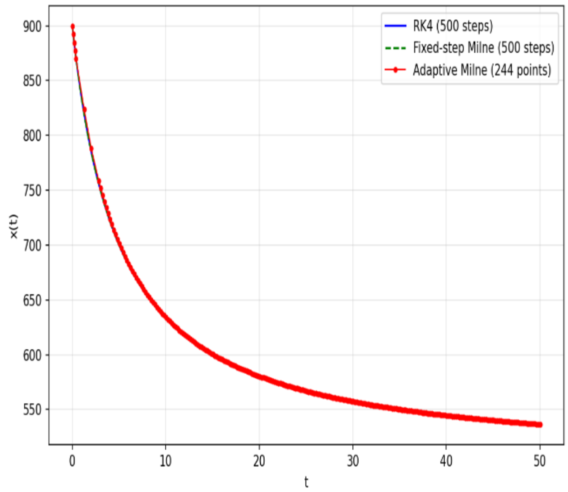

Figure 2. Compares the population trajectories obtained using RK4, the fixed-step Milne method, and the proposed adaptive Milne method under critical harvesting conditions.

Above-Critical Harvesting Regime (H = 150)

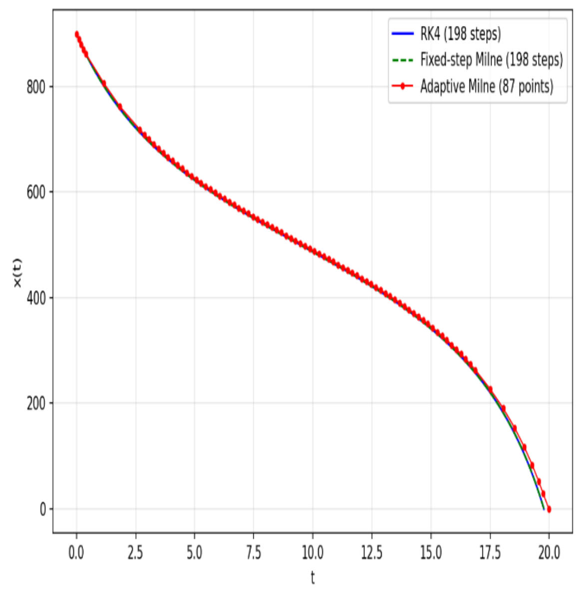

Figure 3. Compares the population trajectories obtained using RK4, the fixed-step Milne method, and the proposed adaptive Milne method under high harvesting conditions.

Adaptive step-size history

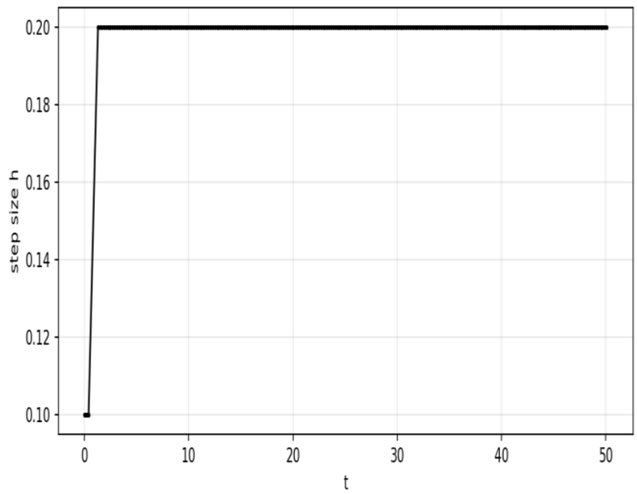

Figure 4. Adaptive step-size history — low harvesting (H = 50).

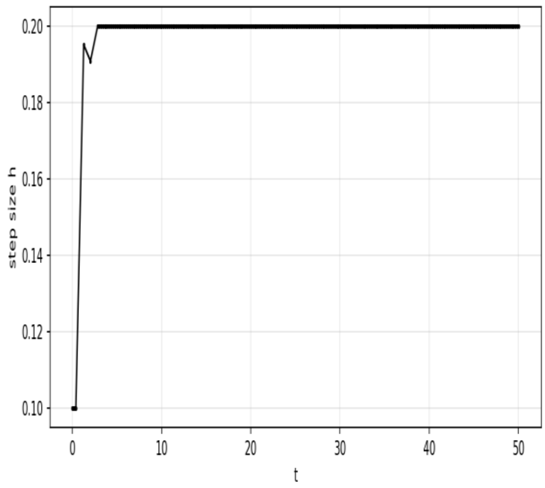

Figure 5. Adaptive step-size history — critical harvesting (H = 125).

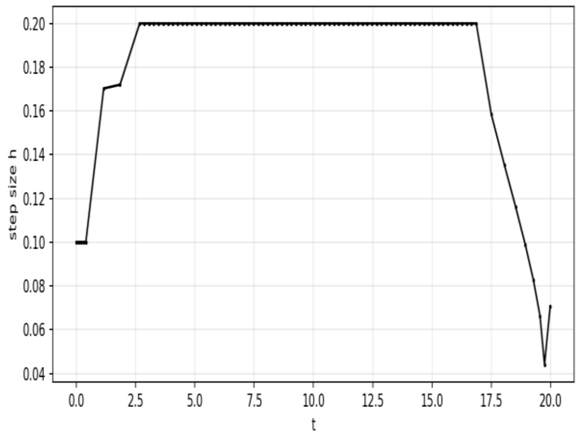

Figure 6. Adaptive step-size history — high harvesting / collapse (H = 150).

Low harvesting regime (H= 50)

In Figure 1, the three numerical solutions overlap almost perfectly throughout the simulation interval, indicating that all methods accurately capture the stable dynamics of the harvested logistic model. The population increases gradually before approaching the theoretical stable equilibrium predicted by the analytical model. No noticeable deviation is observed among the three methods, confirming that the adaptive strategy preserves the fourth-order accuracy of the classical schemes. The corresponding adaptive step-size history is presented in Figure 4.4. During the initial transient phase, relatively small step sizes are adopted to resolve the rapidly changing solution accurately. As the population approaches equilibrium and the rate of change decreases, the adaptive algorithm progressively increases the step size. Consequently, the adaptive Milne method achieves approximately a 50% reduction in the total number of integration steps while maintaining the same level of numerical accuracy as the fixed-step methods.

Critical harvesting regime (H=125)

The numerical solutions obtained at the critical harvesting rate are illustrated in Figure 4.2. At this threshold, the population approaches the critical equilibrium much more slowly than in the previous scenario. All three numerical methods remain in close agreement throughout the simulation, demonstrating that they accurately reproduce the slow transient behaviour associated with the maximum sustainable yield condition. The absence of noticeable numerical oscillations indicates that the adaptive algorithm maintains both stability and accuracy even when the solution evolves gradually over an extended period. The corresponding adaptive step-size history is shown in Figure 4.5. Unlike the low harvesting case, the adaptive algorithm maintains moderate step sizes over most of the simulation because the solution changes slowly near the critical equilibrium. This behaviour enables the solver to achieve an effective balance between computational efficiency and numerical accuracy.

Above-Critical Harvesting Regime (H = 150)

The behaviour of the numerical methods under excessive harvesting is presented in Figure 4.3. When the harvesting rate exceeds the critical threshold, the population declines continuously until extinction. All three numerical methods predict essentially the same extinction trajectory and extinction time, confirming their ability to reproduce the theoretical behaviour of the harvested logistic model. However, the adaptive Milne method automatically refines the integration step during the rapid decline phase, allowing the extinction event to be captured accurately without requiring a uniformly small step size throughout the entire simulation. The corresponding adaptive step-size history is presented in Figure 4.6. Initially, relatively large step sizes are employed while the population changes slowly. As extinction is approached and the solution varies more rapidly, the adaptive algorithm automatically reduces the step size to preserve numerical accuracy. This adaptive refinement enables accurate prediction of extinction time while substantially reducing the overall computational effort.

Results for Parameter Set II

The second parameter set was selected to evaluate whether the comparative performance of the numerical methods remains consistent for populations characterised by higher intrinsic growth rates and larger carrying capacities. This scenario provides a more demanding numerical test because rapidly changing solutions often increase the likelihood of instability in fixed-step multistep methods.

Low Harvesting Regime (H = 600)

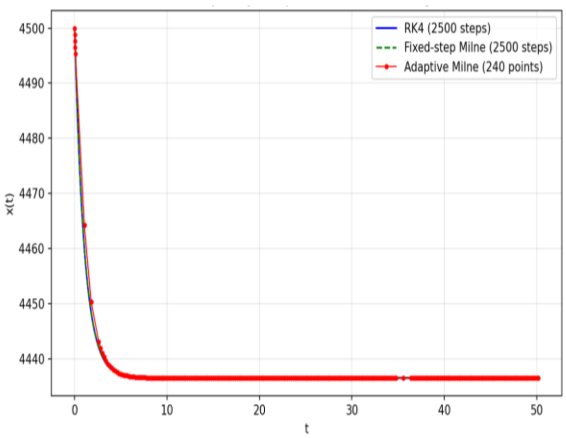

Figure 7. Compares the population trajectories obtained using RK4, the fixed-step Milne method, and the proposed adaptive Milne method under low harvesting conditions.

Critical harvesting H = 1500

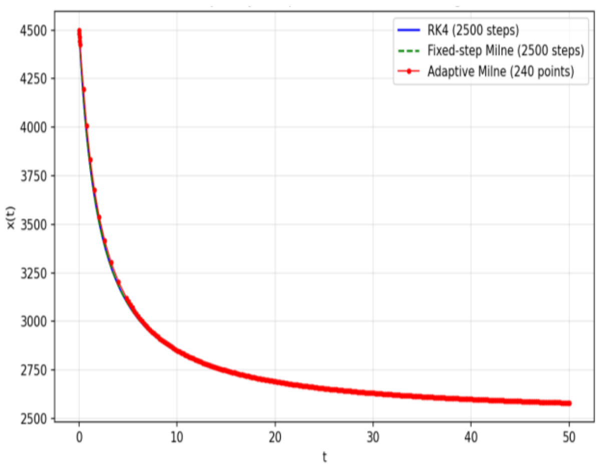

Figure.8. Population trajectories obtained using RK4, the fixed-step Milne method, and the proposed adaptive Milne method under critical harvesting conditions.

High harvesting regime H = 1800

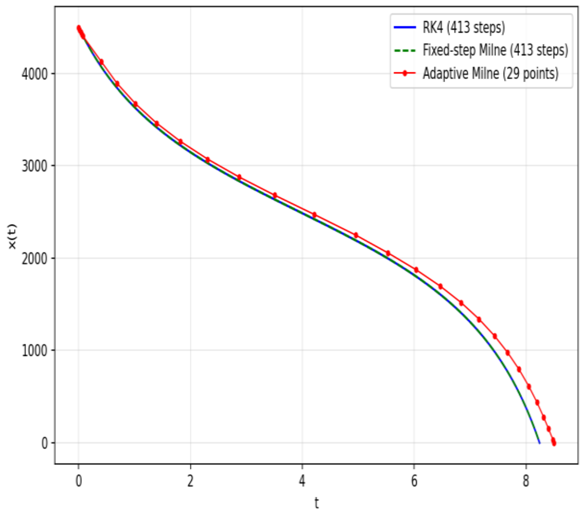

Figure 9. Compares the population trajectories obtained using RK4, the fixed-step Milne method, and the proposed adaptive Milne method under high harvesting conditions.

Adaptive Step-Size History

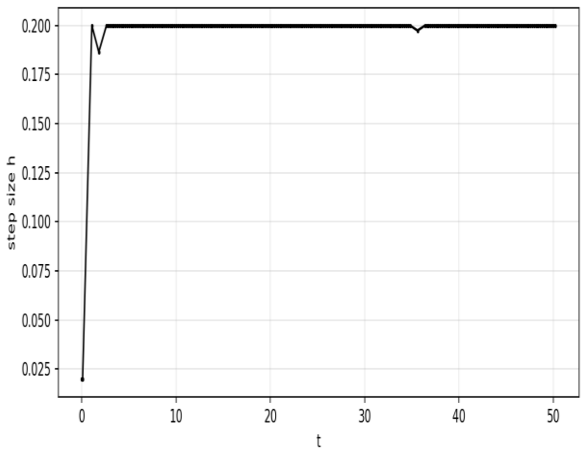

Figure 10. Scenario 2 adaptive step-size history — low harvesting (H = 600).

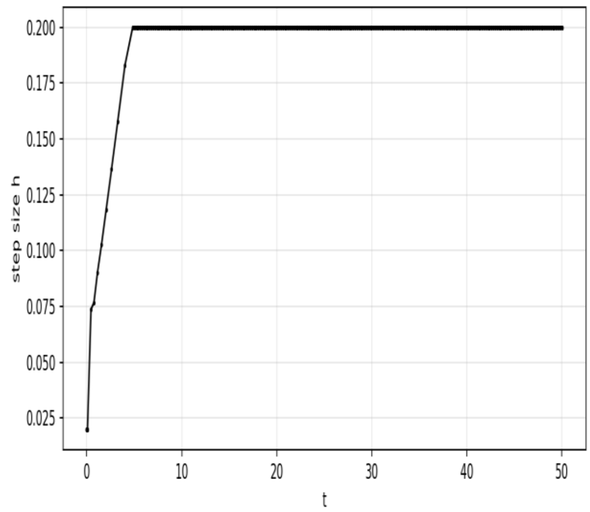

Figure 11. Scenario 2 adaptive step-size history — critical harvesting (H = 1500).

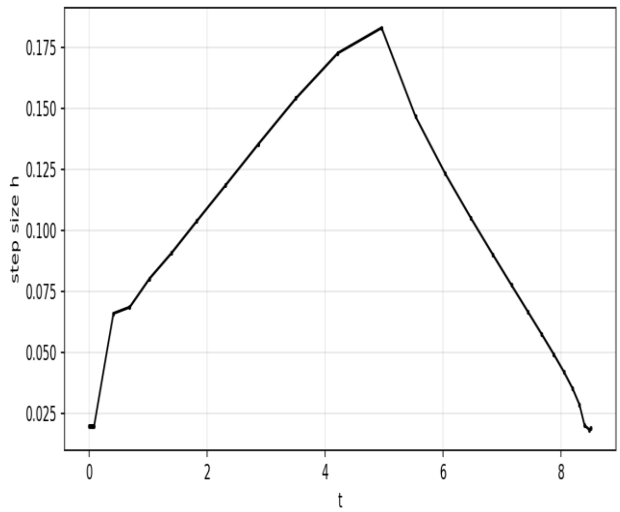

Figure 12. Scenario 2 adaptive step-size history — high harvesting / collapse (H = 1800).

Low Harvesting Regime (H = 600)

The numerical trajectories shown in Figure 4.7 demonstrate that all three methods accurately converge to the stable equilibrium. The numerical solutions remain virtually indistinguishable throughout the simulation interval, confirming that the adaptive Milne method maintains the same level of accuracy as RK4 and the fixed-step Milne scheme. The adaptive step-size history presented in Figure 4.10 indicates more frequent adjustments than those observed for Parameter Set I. The higher intrinsic growth rate requires the adaptive algorithm to respond more actively to changing solution behaviour while still maintaining computational efficiency.

Critical Harvesting Regime (H = 1500)

The numerical results for the critical harvesting case are shown in Figure 4.8. All three methods continue to produce nearly identical solutions, demonstrating that the adaptive Milne method remains reliable even under more demanding dynamic conditions. The corresponding adaptive step-size history in Figure 4.11 shows that the algorithm continuously adjusts the integration step to maintain the prescribed error tolerance while avoiding unnecessary computations.

Above-Critical Harvesting Regime (H = 1800)

The trajectories presented in Figure 4.9 illustrate the collapse of the population when harvesting exceeds the critical threshold. Although all three numerical methods successfully predict population extinction, the adaptive Milne method demonstrates superior computational efficiency by allocating smaller step sizes only during periods of rapid solution variation. The adaptive step-size history shown in Figure 4.12 confirms that the algorithm automatically concentrates computational effort near the extinction phase, where higher numerical resolution is required. This behaviour effectively suppresses the weak step-size-dependent instability associated with the conventional fixed-step Milne method, particularly for rapidly growing populations.

Discussion

Across both parameter sets and all harvesting scenarios, the adaptive Milne predictor–corrector method consistently produced numerical solutions that were in excellent agreement with those obtained using RK4 and the conventional fixed-step Milne method. These results demonstrate that introducing adaptive step-size control does not compromise numerical accuracy.

The principal advantage of the proposed method lies in its computational efficiency. By automatically increasing the integration step during slowly varying solution intervals and reducing it only when rapid changes occur, the adaptive algorithm substantially decreases the total computational cost while preserving solution accuracy. This advantage becomes particularly evident during long-term simulations and near extinction events, where fixed-step methods either require unnecessarily small step sizes or become susceptible to numerical instability.

Overall, the numerical experiments demonstrate that the adaptive Milne predictor–corrector method provides a reliable, efficient, and stable numerical framework for solving harvested logistic growth models across a wide range of ecological conditions.

CONCLUSION

This study developed and evaluated an adaptive Milne predictor–corrector method for the numerical solution of the harvested logistic growth model. The proposed method incorporates local error estimation and automatic step-size control to improve computational efficiency while maintaining the numerical accuracy expected of fourth-order integration schemes. Its performance was assessed against the classical fourth-order Runge–Kutta (RK4) method and the conventional fixed-step Milne predictor–corrector method using two biologically realistic parameter sets under low, critical, and above-critical harvesting regimes.

The numerical experiments demonstrated that the adaptive Milne method consistently produced solutions that were in excellent agreement with those obtained using RK4 and the fixed-step Milne method across all simulation scenarios. The close agreement of the numerical trajectories confirms that the adaptive strategy preserves solution accuracy while providing a more efficient integration procedure. A major advantage of the proposed approach is its ability to adjust the integration step automatically according to the local behaviour of the solution. Larger step sizes were adopted during slowly varying phases of population growth, whereas smaller step sizes were introduced only when rapid changes occurred, particularly near extinction. Consequently, the adaptive method achieved substantial reductions in computational effort without compromising numerical reliability.

The study also showed that the adaptive Milne method effectively mitigates the weak step-size-dependent instability associated with the conventional fixed-step Milne predictor–corrector scheme, especially for populations with relatively high intrinsic growth rates. This improved stability enables the method to provide accurate predictions of equilibrium behaviour, critical harvesting thresholds, and extinction times under a wide range of ecological conditions. These characteristics make the adaptive approach particularly suitable for long-term ecological simulations and repeated scenario analyses, where both computational efficiency and numerical robustness are essential. Overall, the findings demonstrate that adaptive multistep integration provides a practical and reliable alternative to traditional fixed-step numerical methods for harvested population models. The proposed method has potential applications in fisheries management, wildlife conservation, forestry, and other renewable resource systems where rapid and dependable simulation is required to support sustainable management decisions. Future research may extend the proposed algorithm to more complex ecological models, including systems with time-dependent harvesting, stochastic environmental effects, predator–prey interactions, age-structured populations, and spatially distributed ecosystems. Such extensions would further evaluate the versatility and scalability of the adaptive Milne predictor–corrector framework for solving increasingly realistic population dynamics models.

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