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Modeling Temperature Forecast in Ogun State Nigeria with SARFIMA and SARIMA

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Abstract

This studies aimed to assess the forecasting capabilities of Seasonal Autoregressive Integrated Moving Average(SARIMA) and Seasonal autoregressive fractional integral moving average (SARFIMA) models in modelling the weather prediction of Ogun State, Nigeria. The results indicate that the SARFIMA model outperforms SARIMA in terms of fit, serial correlation analysis, and accuracy measures. Forecast validation statistics confirmed the efficacy of the SARFIMA model, as demonstrated by various validation tools. Out-of-sample forecasts for 2019 to 2028 predict a steady rise in temperature, particularly in the Ijebu Ode axis compared to the Abeokuta region. This temperature increase suggests that climate change could significantly impact the livelihoods and economic sectors of Ijebu Ode and its surroundings if adequate preparations are not implemented.

Keywords: Times series, SARFIMA, SARIMA, Temperature, and forecast

INTRODUCTION

Time series forecasting plays a pivotal role in meteorology and environmental applications, encompassing variables like humidity, rainfall, temperature, stream flow, and more. This technique relies on historical data to construct the most suitable model for predicting future values. As described by Raicharoen (2003), it involves utilizing past data to forecast forthcoming values accurately. Weather forecasts are formulated by gathering information about the current state of the atmosphere within a specific area and leveraging this knowledge to predict atmospheric changes. Temperature exerts undeniable effects on various aspects of the environment, agriculture, water consumption, and human activities, as noted by Sarraf et al. (2011). Additionally, it influences nearly all other climatic variables, including relative humidity, evaporation rate, wind direction, wind speed, and precipitation patterns.

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However, providing precise forecasts of air temperature is challenging due to its complex and chaotic nature.

Various researchers, including Murat et al. (2018), Jibril and Sanusi (2019), Adams and Bamaga (2020), Nnoka et al. (2020), Amjad et al. (2023), Adewole (2023), and others, have conducted studies on modelling meteorological variables in diverse locations using time series analysis. Over time, scholars have introduced numerous time series models in the literature to enhance the effectiveness and accuracy of time series modelling and forecasting climate change, both in Nigeria and globally. Among these approaches, the Autoregressive Integrated Moving Average (ARIMA) model is a well-known approach for method for achieving forecasting accuracy and efficiency across various types of time series models. Box and Jenkins introduced an extended ARIMA model known as Seasonal Autoregressive Integrated Moving Average (SARIMA) models, specifically designed for modeling univariate time series data with seasonal components. SARIMA models are proficient at characterizing time series that exhibit non-stationary behaviors both within and across seasons (Box and Jenkins, 1976). Many time series data observations demonstrate long memory, prompting the development of methodologies capable of estimating and predicting autocorrelation functions that decay slowly to zero. A series displaying a fractionally integrated pattern is typified by a stable average sequence of long swings. This phenomenon is observed through the Autocorrelation Function (ACF) declining very slowly over time (Granger !980)). The Autoregressive Fractionally Integrated Moving Average (ARFIMA) model, introduced by Granger and Joyeux (1980), is a fractional order model technique that extends conventional integer-order models such as Autoregressive Integrated Moving Average (ARIMA) and Autoregressive Moving Average (ARMA) models. Additionally, the Seasonal Autoregressive Fractionally Integrated Moving Average (SARFIMA) model, introduced by Porter-Hudak (1990) expands upon ARFIMA models to address both short and long memory components of seasonal variations.

RELATED STUDIES

Researchers have demonstrated the feasibility of modeling time series of any size using both SARIMA and SARFIMA estimation methods, as evidenced by studies conducted by Datong & Goltong (2017), Chukwudike et al. (2020), Ubaka et al. (2021), Udo and Shittu (2022), Adewole (2024), among others. This research endeavors to present iterative methods for analyzing, modeling, and comparing the statistical performance of seasonal ARIMA and seasonal fractional ARIMA models for predicting the average annual temperature of Ogun State in Nigeria, with Abeokuta and Ijebu Ode cities serving as case studies. Therefore, this research is crucial as it provides vital information required by meteorologists, agriculturists, and climatologists, aiding decision-makers in their future planning endeavours in Ogun State, Nigeria.

MATERIALS AND METHODS

Study Area and Data Source

Ogun State is located in Southwestern Nigeria within latitudes 6°N and 8°N and longitudes 3°E and 5°E. The state is bounded on the west by the Republic of Benin and on the east by Ondo State. To the north is Oyo state while Lagos State and the Atlantic Ocean are to the south. The state covers about 16,762square kilometer which is approximately 1.81 percent of Nigeria's land mass of about 923,768 square kilometers. The annual average temperature data of Abeokuta and Ijebu ode city in Ogun covering the period of 1990 – 2018 obtained from NIMET (Nigeria metrological Agency) data management unit will be employed for the study.

Seasonal Autoregressive Integrated Moving Average (SARIMA)

A seasonal ARIMA model generally consist of models with seasonal and non-seasonal components of (p, d, q) and (P, D, Q) respectively, it is expressed as; SARIMA (p, d, q)(P, D, Q).

The seasonal component terms of the model are related to the non-seasonal component, but operate with a difference of back shift during the respective season

$$\theta_p(L)\Theta(L^s)(1-L)^d(1-L^s)^D X_t = \emptyset_q(L)\phi_q(L^s)\epsilon_t \quad (1)$$

where $\theta_p(L) = 1 - \Theta_1 L - \Theta_2 L^2 -, \ldots, \Theta_p L^p$ (2

$$\Theta_p(L^s) = 1 - \Theta_p L^s, \ldots, \Theta_p L^{ps} \quad (3)$$

$$\emptyset_q(L) = 1 + \emptyset L + \emptyset L -, \ldots, + \emptyset_q L^q \quad (4)$$

$$\phi((L^s) = 1 + \emptyset_1 L^s + \emptyset L^2 -, \ldots, + \emptyset L^{qs} \quad (5)$$

where L represents the non-seasonal backshift operators and d is the non-seasonal differencing order. For seasonal part, Θ_p is the seasonal AR component coefficients while θ_q is the seasonal moving Average component coefficients, L^s is the seasonal backshift operators and D representing the seasonal differencing order.

Box Jenkins Methods of SARIMA Model

Identifying a perfect Seasonal ARIMA model for a specific time series analysis, Box and Jenkins (1970) projected a procedure that consists of four major steps, namely,

i) Identification of the model: discovering a tentative model by cshecking the stationarity of the data.

ii) Parameters of the model estimation: Estimating the coefficients of the models by maximum likelihood estimation methods.

iii) Checking the goodness of fit of the model: The diagnostic testing of the model involves the normality test (Jarque and Bera, 1980 test), autocorrelation test (Ljung and Box, 1978 statistic), ARCH (squared residuals'

(iv) Utilization of the final model in forecasting

ARFIMA Model Process

The general form of Autoregressive Fractionally Integrated Moving Average (ARFIMA) process is stated as:

$$\theta(\mathrm{L})\,(1-\mathrm{L})^{\mathrm{d}}\mathrm{X}_{\mathrm{t}} = \emptyset(\mathrm{L})\varepsilon_{\mathrm{t}} \qquad (6)$$

where, L is defined as the lag operator such that

$$\mathrm{LX}_{\mathrm{t}} = \mathrm{LX}_{\mathrm{t-1}} \qquad (7)$$

And the $(1-L)^{\mathrm{d}}$ fractional difference operator replaced the usual standard difference operator $(1-\mathrm{L})$ of a short memory SARIMA process, d is a non-integer parameter that represent the level of the fractional difference. ε_{t} is independently and identically distributed with mean 0 and variance σ^2, $\theta(L)$ and $\emptyset(L)$ signify AR and MA components respectively. The method is covariance stationary for the range of - 0.5 < d < 0.5; involving mean reversion when d <1. Granger (1980), Granger and Joyeux (1980), and Hosking (1981) works described ARFIMA process as a generalized fractional white-noise process.

SARFIMA (*p*,*d*,*q*)× (P,D,Q)s Process

A special formulation of the generalized ARFIMA model was considered by Porter-Hudak (1990). This formulation enables the reproduction of long memory periodicity from short memory in the autocorrelation function of the process, the general form of the SAFRIMA model can be defined as ;

Let $\{x_t\}$ represent a stochastic process, then $\{x_t\}_{t\epsilon z}$ is the zero mean,the seasonal autoregressive fractionally integrated moving average process, denoted by SAFRIMA(p,d,q) x $(P,D,Q)_s$ is an extension of the long range dependence in the mean ARFIMA(p, d,q) process, the SAFRIMA(p,d,q) x $(P,D,Q)_s$ process describes time series with long memory or long range dependence or persistent periodical behavior at finite number of spectrum frequencies SAFRIMA(p,d,q) x $(P,D,Q)_s$ process is express as;

$$\theta(\mathrm{L})\Theta(L^s)(1-L)^s(1-L^s)^D x_t = \emptyset(\mathrm{L})\phi(L^s)\varepsilon_{\mathrm{t}} \quad \text{for } t\epsilon z \qquad (8)$$

where $s\epsilon N$ denotes the seasonal period, L represents the backward shift operator, $(1-L^s)^D$ is the seasonal difference operator $\Theta(\cdot)$ and $\phi(\cdot)$ and are the polynomials of degrees P and Q, respectively, defined by:

$$\Theta(L^s) = \sum_{i=0}^{P}(-\Theta_i)L^{si} \qquad (9)$$

$$\phi(L^s) = \sum_{j=0}^{Q}\left(-\phi_j\right)L^{sj} \qquad (10)$$

where$\Theta_i$and ϕ_j are constants. The seasonal difference operator $(1-L^s)^D$, with seasonality $s\epsilon N$ for all $D > -1$, is defined by means of the binomial expansion;

$$(1-L^s)^D = 1 - DL^s - \frac{(D(1-DL^{2s})}{2!} - \frac{(1-D(2-D)L^{3s})}{3!} -, \ldots, \qquad (11)$$

Assume that $\theta_p(L)\Theta_p(L^s) = \emptyset_q(L)\phi_q(L^s) = 0$ in equation (1) above has no common zero, then the following criteria hold for SAFRIMA model;

a)The stochastic process $\{x_t\}$ is stationary if $d + D < 0.5$, $D < 0.5$and $\theta_p(L)\Theta(L^s) \neq 0$ for $|B| \leq 1$.

b). The stationary process $\{x_t\}$ has a long memory property if $0 < d + D < 0.5, 0 < D < 0.5$ and $\theta_p(L)\Theta_p(L^s) \neq 0$ for $|B| \leq 1$.

c). The stationary process $\{x_t\}$ has an intermediate memory property if $-0.5 < d + D < 0, -0.5 < D < 0$ and $\theta_p(L)\Theta_p(L^s) \neq 0$ for $|B| \leq 1$.

d). The series; $\{x_t\}$ is non-stationary if $0.5 \geq d + D < 1$.

SARFIMA model allows times series to be fractionally integrated, it generalize the integer order of SARIMA model integration in allowing the difference parameter to take on fractional values If a series exhibits long memory, it is neither stationary (I (0)) nor is it a unit root (I (1)) process; the series is an I(d) process.

Pre-Estimation process
Long Memory Test

One of the preliminary steps in estimating SARFIMA models is to determine whether the observed data series exhibits long memory behavior. This can be assessed using the Hurst Exponent technique to check if the data conforms to long memory structures.

Hurst Exponent

The Hurst exponent is one of the time series long-memory families. The long memory structure happens when the values of H fall in the interval $0.5 < H < 1$. The Hurst exponent estimation process uses the formula:

$$H = \frac{\log\left(\frac{R}{S}\right)}{\log(N)} \qquad (12)$$

N signifies length of the sample data and $\frac{R}{S}$ is the matching value of the rescaled evaluation.
Techniques of augmented Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests will be employed to investigate stationarity of the meteorological data in Ogun state and fractional integration modelling.

Augmented Dickey Fuller Test of Stationarity: ADF test model is expressed as;

$$\Delta X_t = \alpha X_{t-1} + Y_t\varphi + \beta_1\Delta X_{t-1} + \beta_2\Delta X_{t-2} + , \ldots , \beta_p\Delta X_{t-p} \qquad (13)$$

where,
ΔX_t represents the differenced series
ΔX_{t-1} is the immediate past observations.
Y_t signifies the optional exogenous regressor which is either a constant or a constant trend
α and φ are parameters needed to be estimated.
$\beta_1, \ldots, \beta_p$ denotes the coefficients of the lagged terms.

The ADF test statistic is expressed as;

$$t_\alpha = \frac{\hat{\alpha}}{S_e(\hat{\alpha})} \qquad (14)$$

The test of hypothesis involves;
$H_0{:}\alpha = 0$, it infers that the series has unit roots
$H_1{:}\alpha < 0$, it infers that the series has no unit roots.
Decision rule: Reject H_0: if t_α is less than asymptotic critical value

Kwiatkowski-Philips-Schmidt-Shin (KPSS)Test

The KPSS test of stationarity was developed by Kwiatkowski et al (1992). The null hypothesis assumes that the Data Generating Process (DGP) is stationary. Considering the following DGP without a linear trend;

$$y_t = x_t + z_t \qquad (15)$$

where

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + , \ldots , \alpha_p x_{t-p} + u_t \qquad (16)$$

$u_t \sim iid(0.\sigma^2)$ and z_t is assume to follow a stationary process.

KPSS test statistic is given as;

$$KPSS = \frac{1}{T^2}\sum_{t=1}^{T}\frac{s_t^2}{\sigma^2 p} \qquad (17)$$

where $s_t = \sum_{j=1}^{t}\hat{m}_j$ with $\hat{m}_t = x_t - x$ and $\hat{\sigma}_p{}^2$ is an estimator of the long run variance of the stationary Process z_t.

SARFIMA Process Estimation
Estimation of Fractional Difference Parameter

The long memory parameter can be estimated using three major approaches: non-parametric, semi-parametric, and parametric methods. This research will focus exclusively on the semi-parametric method.

Semi-parametric Method

Semi-parametric method of estimating d in the frequency domain proposes by Geweke and Potter-Hudak (1993). This method considers the power spectrum of the ARFIMA(p, d, q) process, $\{x_t\}$ given as,

$$f_X(w) = |1 - e^{-iw}|^{-2d} f_z(w) \qquad (18)$$

Where $f_X(w)$ and $f_z(w)$ are the spectral densities of x_t and x_z respectively, can be simplified as;

$$ln\,[f_X(w)] = -[\quad d ln[4sin^2(w/2)] + ln f_z(w) \qquad (19)$$

$$ln\,[f_X(w_t)] = ln\,[f_z(w_t = o)] - d ln[4sin^2(w_t/2)] + ln\,[f_z(w_t)] - ln\,[f_z(w_t = o)] \qquad (20)$$

In forms of regression equation, equation (21) becomes

$$ln\,[f_X(w_t)] = a + bx_t + \varepsilon_t \qquad (21)$$

where

$$a = ln\,[f_z(w_t = o)] \qquad (22)$$

$$x_t = \ln\left[4sin^2\left(\frac{w_t}{2}\right)\right] \qquad (23)$$

$b = -d$

$\varepsilon_t = \{ ln\,[f_z(w_t)] - ln\,[f_z(w_t = o)]$ is the error in the model for t = 1,2, ⋯ ,n.

Post Estimation Process

Model Selection

Optimum selection criteria were employed in model selection by selecting the model with minimum Akaike Information Criteria (AIC) and Schwarz Information Criterion (SIC)

Model Diagnostic

To validate the appropriateness of the selected SARIMA and SARFIMA models, the white noise, serial correlation, and heteroscedasticity were evaluated using the residual normality test, the Portmanteau test, and the Autoregressive Conditional Heteroscedasticity Lagrange Multiplier (ARCH-LM) test, respectively. This involved examining the hypothesis that the residuals are white noise, assumed to be independently distributed.

Employing the methods of Ljung and Box (1978), The estimated autocorrelations of residuals ρk, k=1,2,…,K are validated via a chi-squared statistic:

$$N(N+2)\sum_{k=1}^{K}\frac{[\rho_k(\varepsilon)]^2}{N-K} \approx \chi^2\,(K-1) \quad (24)$$

$$\rho_k(\varepsilon) \approx N(0,1)$$

where K-1 = k-p-q, N is the sample size and ρ symbolize the autocorrelation coefficient.

Model Forecasting and Performance Evaluation

The predicting performance of selected models is assessed via various validation criterions such as Akaike Information criteria (AIC) and Schwarz Information Criterion (SIC)

$$AIC = 2T - m \quad (25)$$

$$SIC = 2T\log n - \log m \quad (26)$$

T represents the total number of estimable parameters, m denotes the maximum likelihood, and n is the number of samples. Additionally, the forecast accuracy of the SARFIMA and SARIMA models is evaluated using the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE), and the Mean Absolute Percentage Error (MAPE).

MAE symbolizes the absolute difference between the forecasted values and the actual values. It estimates the average absolute deviation of predicted values from real values. The MAE is calculated as follows;

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\hat{\hat{y}}_f - y_t\right| \quad (27)$$

MAPE is projected as the mean absolute percent error for each time period minus real values divided by real values. It computes the percentage of mean absolute error that occurred in the model formation. It is given as;

$$MAPE = \frac{100}{n}\sum_{t}^{n}\left|\frac{\hat{\hat{y}}_f - y_t}{y_t}\right| \quad (28)$$

RMSE explicate the absolute fit of the model to the observed data, it is figured as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\hat{\hat{y}}_f - y_t} \quad (29)$$

$\hat{\hat{y}}_f$ and y_t represent the estimated and the real values respectively; n is the sample size, model with smaller criteria values is preferred for best superior forecasting precision.

RESULTS AND DISCUSSION

Table 1. Descriptive Statistics

	Mean	**Maxi Mum**	**Mini mun**	**Std. Dev.**	**Med.**	**Skew Ness**	**Kurt osis**	**Jarque Bera**	**P. Val.**	**Obs.**
ABEOKUT	132.44	149.83	114.12	7.52	131.33	0.046	3.23	0.079	0.96	29
I- ODE	279.00	285.36	273.02	3.60	278.68	0.096	-0.96	0.341	0.02	29

Table 1 gives the summary of average annual temperature data in Abeokuta and Ijebu Ode from 1990 to 2018, Ijebu Ode reports a hotter temperature than Abeokuta. The series are normally distributed for as indicated by the low Jarque-Bera test values and high p-value.

Table 2. Stationarity test results at level.

	ADF		PP		KPPS	
Variables	**ADF Test Stat**	**Prob.**	**PP Test Stat.**	**Prob.**	**KPSSTest Stat.**	**Prob.**
ABEOKUTA	-3.0031	0.015	-2.731*	0.019	-3.6820	0.0000
IJEBU ODE	0.2741	0.173	0.061	0.2802	-5.219	0.0000

*Note * indicate significance at α = 0.05 at level.*

The various stationarity tests at level are presented in Table 3. The tests showed that the average annual series shows nonstationary features. Moreover Mann–Kendall (MK) test in Table 3 also established a trend in the data series reinforcing non-stationarity

Table 3. Seasonal Mann Kendall Trend Analysis

Parameters/ City	**ABEOKUTA**	**IJEBU ODE**
Kendall's tau	0.3282*	0.1623*
Sen's Slope	0.4524	0.1118
S	136	118
P value	0.0014	0.0003

*Note * indicate significance at α = 0.05*

The SARIMA procedure requires the series to meet stationarity and invertibility conditions for accurate modeling (Nury et al., 2013). Non-stationarity in the series was addressed by differencing the data to achieve stationarity. The results of the stationarity test at the first difference are presented in Table 4.

Table 4. Stationarity test results at First Difference.

	ADF		PP		KPPS	
Variables	**ADF Test Stat**	**Prob.**	**PP Test Stat**	**Prob**	**KPSS Test Stat.**	**Prob.**
ABEOKUTA	-2.943	0.221	-0.3913	0.000	-3.837	0.000
IJEBU ODE	0.6411	0.128	0.0013	0.292	-2.179	0.000

*Note * indicate significance at α = 0.05 in first difference. Moreover, from Table 4 above, KPSS confirmed the stationarity of the annual data series.*

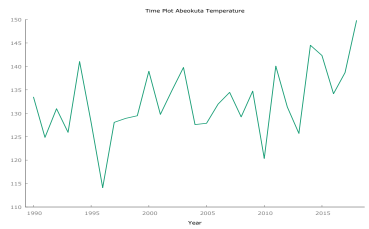

Figure 1. ABEOKUTA Ave. Annual Temp

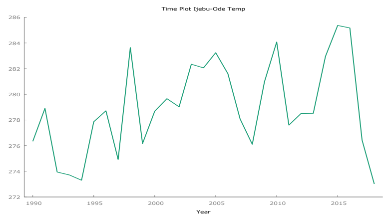

Figure 2. IJEBU ODE Ave. Annual Temp.

Figure 1 and 2, express the seasonality of the average annual temperature data for Abeokuta and Ijebu Ode respectively. Fig 3 and 4 below presents the correlogram plot of the Abeokuta and Ijebu Ode average annual temperature series at first difference.

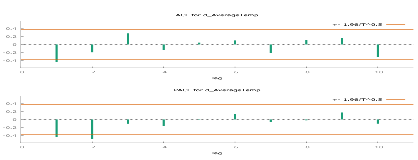

Figure 3. corellogram of ABEOKUTA series

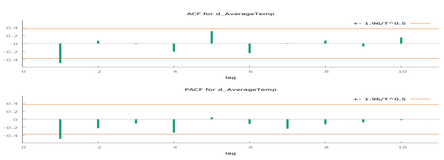

Figure 4. corellogram of IJEBU ODE series

Seasonal ARIMA Model Result

Table 5. Seasonal ARIMA Model

Model/CIT	ABEOKUTA			IJEBU ODE		
Model	**(p,d,q)(P,D,Q)s**	**AIC**	**SIC**	**(p,d,q)(P,D,Q)s**	**AIC**	**SIC**
Model 1	**SARIMA (1,1,1) (1,1,1)12**	**10.005**	**10.175**	SARIMA (1,1,1)(1,1,0)12	7.334	7.429
Model 2	SARIMA (1,1,2)(1,1,0)12	10.108	10.216	**SARIMA (1,1,1) (1,1,1)12**	**7.218**	**7.305**
Model 3	SARIMA (2,1,1)(1,1,1)12	10.274	10.339		7.440	7.483

The correlograms help in obtaining the various Seasonal ARIMA fitted to the series presented in Table 5 above, the models with the lowest AIC and SIC values were selected as the best among the competitors. The best model is highlighted in bold for easier identification.

Table 6. Parameter Estimates of the Seasonal ARIMA fitted model

	ABEOKUTA			IJEBU ODE		
Par.	**Coeff.**	**St Error**	**Prob.**	**Coeff.**	**St Error**	**Prob.**
θ_1	0.1244	0.0713	0.0035	0.4935	0.3040	0.0055
Θ_1	0.3872	0.0311	0.0017	0.2118	0.0266	0.0000
$\emptyset_1$	0.5215	0.0813	0.0026	0.1893	0.0164	0.0035
Φ_1	0.0328	0.0480	0.0009			

Table 6 gives the parameter estimates of the SARIMA fitted model of Average Annual Temperature of both Abeokuta and Ijebu ode based on the selection criteria in Table 5. Parameter θ_1 is the autoregressive parameters of non- seasonal components, Θ_1 is the moving average parameters of non - seasonal components, $\emptyset_1$ is the autoregressive parameters of seasonal component and ϕ_1 is the moving average parameters of the seasonal. The parameter estimation engaged maximum likelihood method of estimations adopted from Box and Jenkins procedures. Table 7 below reports the diagnostics evaluation of SARIMA Models. The selected models are normally distributed; residuals are homoscedastic in nature and no serial correlation

Table 7. Statistical tests of the residuals of selected SARIMA models.

Times series	SARIMA	Autocorrelation Test		Heteroskedacity Test		Normality test	
	Model	**Lung Box Q.**	**Portmanteas**	**Breusch Pagan**	**White**	**Jarque Bera Test**	**Shapiro Wiki**
		Prob.	**Prob.**	**Prob.**	**Prob.**	**Prob.**	**Prob.**
ABEOKUTA							
AVE.TEMP	**SARIMA(1,1,1)(1,1,1)12**	0.2289	0.326	0.3127	0.4167	0.3390	0.2415
IJEBU ODE							
AVE. TEMP	**SARIMA(1,1,1)(1,1,1)12**	0.156	0.2302	0.2781	0.3301	0.1903	0.2891
α		0.05	0.05	0.05	0.05	0.05	0.05

Seasonal Autoregressive Fractionally Integrated Moving Average Process (SARFIMA Model)

Table 8. Long Memory tests of the SARFIMA models.

	ABEOKUTA	IJEBU ODE
HURST.E /RS	0.8622 (0.007)	0.7942 (0.000)

Note: Hurst. E/ RS is the Hurst Exponent Rescaled Range.

The existence of long memory in the series is confirmed in Table 8 above through the Hurst exponent values obtained using the rescaled Range method. Table 9 presents the estimates of the fractional difference of the average annual temperature for both Abeokuta and Ijebu-Ode, utilizing an automatic initialization of the integration with Geweke and Porter-Hundlak log-periodogram regression. Table 9 provides a tabulation of the competitively estimated models for each series and their corresponding values for the selection criteria. The best model for each series is highlighted in bold print and marked with an asterisk

Table 9. SARFIMA Model

Model/ CTY	ABEOKUTA					IJEBUODE				
Model	**$(p,d,q)(P,D,Q)_s$**	**D**	**D**	**AIC**	**BIC**	**(p,d,q)(P,D,Q)**	**D**	**D**	**AIC**	**SIC**
Model 1	SARFIMA $(1,d,1)(1,D,0)_{12}$	0.2246	0.1945	10.044	10.520	SARFIMA $(2,d,1)(1,D,1)_1$	0.642	-0.12	5.569	5.802
Model 2	SARFIMA $(2,d,1)(1,D,1)_{12}$	0.4689	-0.5632	12.569	12.731	**SARFIMA $(2,d,2)(0,D,1)_1$**	**0.376**	**0.512**	**5.137**	**5.297***
Model 3	**SARFIMA $(1,d,1)(2,D,1)_{12}$**	**0.6643**	**0.8531**	**9.729***	**9.918***	SARFIMA $(1,d,0)(1,D,1)_1$	- 0.064	0.332	6.149	6.382
Model 4	SARFIMA $(0,d,1)(1,D,0)_{12}$	0.4832	0.3521	11.229	11.416	SARFIMA $(2,d,1)(0,D,1)_1$	0.558	- 0.642	7.337	7.404

Table 10. Parameter Estimates of the SARFIMA fitted model of Average Annual Temperature

	ABEOKUTA			IJEBU ODE			
Par.	**Coeff.**	**St Error**	**Prob.**	**Par.**	**Coeff.**	**St Error**	**Prob.**
D	0.6643	0.3791	0.0036	D	0.3762	0.0215	0.0003
D	0.8531	0.5316	0.0246	D	0.5121	0.4140	0.0066
θ_1	0.0163	0.2526	0.0000	θ_1	0.3273	0.1766	0.0271
Θ_1	0.2642	0.4291	0.0000	θ_2	0.6542	0.2854	0.0032
$\emptyset_1$	0.2854	0.4442	0.0134	Θ_1	0.3874	0.3003	0.0105
$\emptyset_2$	0.0698	0.5616	0.0022	Θ_2	0.8353	0.6286	0.0054
ϕ_1	-0.1715	0.6727	0.0000	ϕ_1	0.5729	0.1935	0.0000

Table 10 gives the parameter estimates of the SARFIMA fitted model of Average Annual Temperature of both Abeokuta and Ijebu ode based on the selection criteria in Table 8. Parameter $\theta_1, \theta_2$are the autoregressive parameters of non- seasonal components, Θ_1, Θ_2 are the moving average parameters of non -seasonal components, $\emptyset_1, \emptyset_2$ are the autoregressive parameters of seasonal component and ϕ_1 is the moving average parameters of the seasonal, d and D represents the fractional difference of non-seasonal and seasonal components respectively

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Diagnostics checks of SARFIMA Models

Table 11. Statistical tests of the residuals of selected SARFIMA models.

Times series	**SARFIMA(p,fd,q)**	**Autocorrelation Test**		**Heteroskedacity Test**		**Normality test**	
	Model	**Lung Box Q**	**Portman teau**	**Breusch Pagan**	**White**	**Jarque Bera Test**	**Shapiro Wiki**
		p- value	**p- value**	**p-value**	**p. value**	**p-value**	**p-value**
ABEOKUTA							
AVE.TEMP	**SARFIMA (1,d,1)(2,D,1)$_{12}$**	0.2173	0.1281	0.2804	0.3912	0.4193	0.2201
IJEBU ODE							
AVE. TEMP	**SARFIMA (2,d,2)(0,D,1)$_{12}$**	0.3392	0.2912	0.3914	0.3105	0.4413	0.5014
α		0.05	0.05	0.05	0.05	0.05	0.05

Table 11 displays the results of evaluating autocorrelation, heteroskedasticity, and normality for each selected SARFIMA model. The normality tests indicate that the residuals generated from the chosen SARFIMA models exhibit a normal distribution. Both the Ljung-Box and Portmanteau values for all variables exceed the significance level, indicating no autocorrelation among the forecast error residuals of the models. Furthermore, the residuals demonstrate homoscedasticity.

Table 12. Evaluation of selected SARIMA and SARFIMA Models forecast Accuracy

	ABEOKUTA				**IJEBU ODE**			
	RMSE	**MAPE**	**MAE**	**R^2**	**RMSE**	**MAPE**	**MAE**	**R^2**
SARIMA	0.3413	0.0196	0.3422	0.8240	0.3641	0.0378	0.4682	0.8935
SARFIMA	0.2947	0.0044	0.2256	0.9062	0.3018	0.0019	0.3978	0.9316

Table 12 displays the forecast accuracy measures for the selected SARIMA and SARFIMA models for the city under study. A comparison of SARIMA and SARFIMA modelling results in Table 12 indicates that the SARFIMA model is more suitable for modelling the average annual temperature of Ogun State. The low values of the unbiased statistic MAPE for SARFIMA models in Table 12 demonstrate the effectiveness of the selected SARFIMA models in accurately predicting the temperature of Ogun State. Additionally, the overall error measures provide evidence of better forecasting performance with SARFIMA models

SARFIMA model Forecast

Forecast values for average annual Temperature of Abeokuta and Ijebu Ode series for the year 2019 to 2028 were presented in Table 13 and 14 with their lower and upper limits respectively employing derived SARFIMA model for the variables.

Table 13. Average Annual Temperature of Abeokuta Out of Sample Forecast using SARFIMA

YEAR	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**
FORECAST VALUES	433.8	426.15	438.19	435.8	432.48	415.14	407.2	347.2	344.2	347.96
Lower Limits	412.6	402.34	428.45	397.7	407.26	423.50	389.1	316.1	315.2	300.20
Upper limits	459.6	461.94	484.15	498.9	519.37	470.20	469.3	477.3	492.5	480.64

Table 14. Average Annual Temperature of Ijebu ode Out of Sample Forecast SARFIMA

YEAR	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**
FORECAST VALUES	311.5	326.37	308.81	321.78	346.18	377.85	383.77	418.66	496.65	484.29
Lower Limits	294.1	310.63	276.35	307.54	238.51	343.74	224.48	367.10	313.91	308.93.
Upper limits	346.8	374.71	379.40	373.31	421.75	532.20	488.06	496.84	545.70	526.44

CONCLUSION

This study analyzed and modeled the annual average temperature of Ogun State, Nigeria, focusing on Abeokuta and Ijebu Ode City as case studies, employing both seasonal autoregressive integrated moving average (SARIMA) and seasonal autoregressive fractional integrated moving average (SARFIMA) processes. The methodology outlined the SARIMA and SARFIMA models, integrating the seasonality of the series. Based on forecast evaluation measures, SARFIMA models demonstrated superior predictive abilities compared to SARIMA for all series. Model appropriateness was confirmed through the normal distribution of residuals and the absence of error autocorrelation. Forecasts were made for a ten-year period, and the forecasted values remained within confidence limits. Results of out-of-sample forecasts from 2019 to 2028 indicate a steady rise in temperature, particularly pronounced in the Ijebu Ode axis compared to the Abeokuta region in Ogun State, Nigeria. This temperature increase suggests ongoing climate change, potentially impacting the livelihoods and economic sectors of Ijebu Ode and its surroundings if there is no adequate preparation.

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